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YERKES ACTINOMETRY

ZONE $+73°$ TO $+90°$

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The problem of the co-ordination of visual and photographic magnitudes of stars has engaged the attention of the writer for some years. An opportunity for work under favorable conditions was presented by the acquisition by the Yerkes Observatory of a Zeiss doublet lens of 14.5 cm aperture and 81.4 cm focal length in 1905; and of an objective prism of the same aperture and 15° angle, in 1906; both of ultra-violet glass. This instrument proved to be extremely well adapted for photometric purposes, so that after some preliminary experiments it was decided to attempt the measurement of the stars of the *Potsdam Photometric Durchmusterung* in the zone from 73° north declination to the pole, and to include: (1) the photographic magnitudes; (2) the "visual" magnitudes (with color-sensitive plates and "visual-luminosity" filter); (3) the resulting "color-indices"; (4) the spectral class on the Harvard system. It was thought better to confine the investigation to the stars down to magnitude 7.5 in a limited zone, rather than to attempt an extended program before the methods of work had been thoroughly tested.

The dependence of photographic photometry on the colors of the stars, and the intimate relation between colors and spectra,

seemed to make the determination of the above list of data of considerable value from the standpoint of astrophysics. A catalogue, including these data, of standard stars conveniently located for observers in the northern hemisphere would also have value as a foundation for further work. This relation between colors and spectra also furnished a useful check on both the spectra and the magnitudes.

ZERO POINT AND SCALE

The choice of zero point and scale is fundamental for a catalogue of stellar magnitudes. Fortunately, the selection of the zero point could be postponed till the completion of the reductions. In the meantime the proposal of the Committee on Magnitudes of the Astrographic Chart[1] gave what seemed to have a chance for acceptance as an "International System," and which was gladly adopted, namely—

> For the stars of spectral type Ao on the Harvard classification, and of magnitude 5.5 to 6.5, the photographic and visual magnitudes shall be assumed equal, and the visual magnitudes shall be reckoned on the Harvard scale.

The determination of an absolute scale for the photographic magnitudes was made possible by the excellent quality of the extra-focal images given by the Zeiss lens. The method was described in detail in a paper by Jordan and the writer,[2] but since the publication of that paper two improvements have been made in the sensitometer, one eliminating internal reflection in the cells, the other providing means of rotating the sensitometer during the exposure. An indirect, but satisfactory check on the scale obtained with the sensitometer, was given by Pleiades plates taken with the 24-inch reflecting telescope and a wire gauze screen, using the "Halbgitter" method suggested by Schwarzschild.[3] The value of the constant b in the equation

$$\text{Magnitude}=a-b\sqrt{D}$$

[1] *Astronomische Nachrichten*, **186**, 40, 1910.

[2] "An Absolute Scale of Photographic Magnitudes of Stars," *Astrophysical Journal*, **26**, 244, 1907.

[3] *Astronomische Nachrichten*, **183**, 297, 1910.

is determined by the scale of the plate. The values found by the two methods were:

	Gingrich
From sensitometer magnitudes....	0.90±0.01
By "Halbgitter" method.........	0.91±0.01

The tabular values used in the reductions are given in the next section with the description of the "Mikrophotometer."

INSTRUMENTS

The camera was made in the shops of the Yerkes Observatory by providing the Zeiss doublet with a brass tube arranged for focusing with a vernier reading to 0.1 mm; a sliding carrier for the plate-holder so that a series of exposures could be placed on the plate; a guiding telescope of 5.3 cm aperture and 90 cm focal length, and an equatorial mounting originally made by Brashear for a 16 cm reflector. The objective prism fits over the cell of the lens, and can be put in place or removed in a few minutes' time. Fig. 1 shows the instrument without the prism.

The excellent quality of the extra-focal images produced by this lens is shown by Fig. 2, a negative of the Pleiades group taken 10 mm outside the focus, giving a practically uniform disk, capable of measurement with great precision. Images taken as near as 6 mm from the focus begin to show slight inequalities, but if the measuring machine is set a little out of focus they can be measured with sufficient accuracy. In practice the plates were taken 6 mm inside the focus, where the field was a little more nearly flat than on the outside. Within an angular distance of 2°5 from the axis of the lens, the illumination of the image is quite uniform, but farther out, the side away from the axis becomes denser, and the measurements, being less precise, were given a smaller weight. No images farther than 3°5 from the axis were measured.

The "Mikrophotometer," with which the extra-focal images were measured, is shown in Fig. 3. A detailed description by the inventor, Hartmann, was published in this *Journal*, **10**, 321, 1899. A single improvement, suggested by Hartmann, was made by mounting the lamp (a 16–20 candle-power incandescent) on a slide, so that a change in the position of the lamp, in a line parallel to the

horizontal microscope, would vary the relative amounts of light sent into the two light-tubes; thus making it possible to bring any given opacity on the plate to a certain reading on the wedge-scale. With the opal glass, furnished with the instrument, covering the junction of the two light-tubes, this adjustment of the

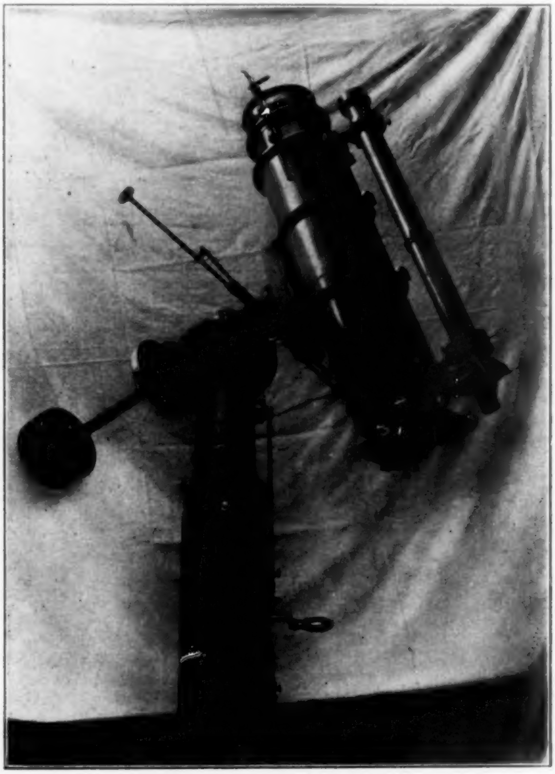


FIG. 1.—Zeiss camera, U-V glass

scale-readings was not possible, since the opal acted as a source of light; but on replacing the opal with a piece of ground glass the adjustment was readily made.

The wedge used in this photometer was one of a photographic set made by King and kindly furnished by Professor E. C. Pickering. Investigations of the visual absorption of these wedges have been

repeatedly made,[1] and indicate a very nearly uniform absorption expressed in magnitudes. For use with photographic plates it was desired to calibrate the wedge directly in terms of stellar magnitude. The sensitometer used contained a nest of 20 light-tight cells, in four rows of five each. The end of the cells exposed to the light (north sky) was covered by a metal plate with a round hole opposite the middle of each cell. Thus the amount of light admitted to each cell was proportional to the area of the hole.

sensitometer

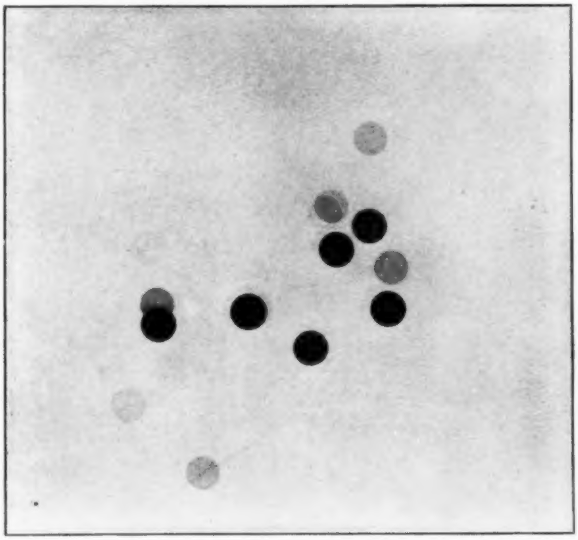


FIG. 2.—Extra-focal images of the *Pleiades* taken with Zeiss camera

The numerical data for the metal plate used (called Plate *D*) are given in Table I.

In this table the diameters of the holes are expressed in millimeters, and the Δ mag. represents the log. diam.[2] (the log. of the relative light) divided by 0.4 to reduce to relative magnitudes. The end of the nest of cells away from the light was covered by a metal plate provided with a hole 1 mm in diameter opposite the center of each cell. The sensitive plate was placed with the film

[1] King, *Harvard Annals*, **41**, 237; Aitkin and Maddrill, *Astrophysical Journal*, **22**, 147, 1905; Parkhurst, *Researches in Stellar Photometry*, pp. 8–10.

in contact with this metal plate, and on exposure to daylight would be impressed by a series of small circles (smaller than the extra-focal star-images, and further distinguished by their regular arrangement) whose opacities would be the same as if caused by a series

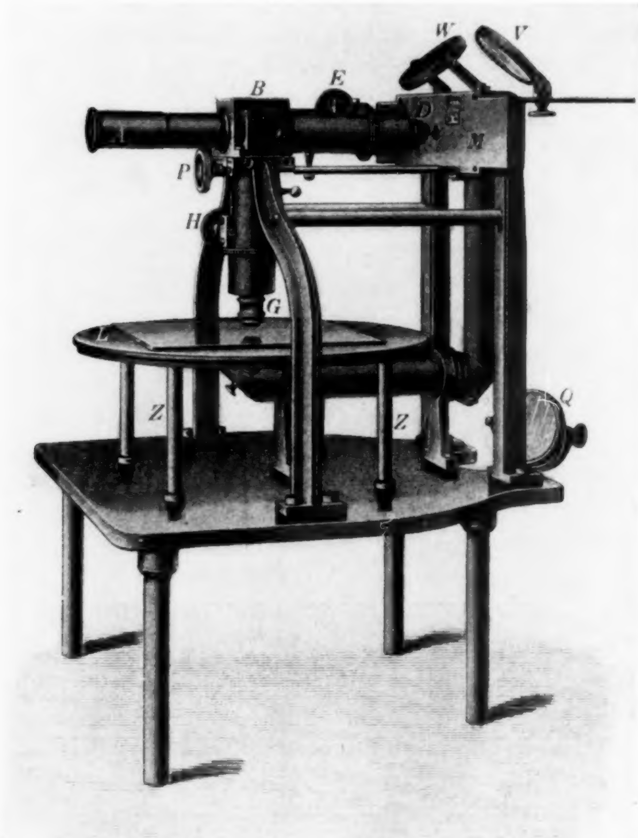


FIG. 3.—Hartmann "Mikrophotometer"

of stars having relative magnitudes represented by the data of Table I. This series was impressed on the zone plate after exposure to the sky and before development, thus giving an absolute scale on the plate.

The machine for measuring diameters of the focal images used, in part of the work, is shown in Fig. 4. The slide carrying the

FIG. 4.—Micrometer for the measurement of disk diameters

plate has rack and pinion motion in two directions. The microscope has a Zeiss position micrometer with two screws at right

TABLE I

Hole	Log. Diam.[a]	Δ Mag.	Hole	Log. Diam.[a]	Δ Mag.
1	0.0198	0.050	11	0.8420	2.105
2	0.0828	0.207	12	0.9024	2.256
3	0.1592	0.398	13	0.9760	2.440
4	0.2360	0.590	14	1.1130	2.782
5	0.3630	0.908	15	1.1508	2.877
6	0.4200	1.050	16	1.2432	3.108
7	0.5134	1.284	17	1.3344	3.336
8	0.5908	1.477	18	1.3972	3.493
9	0.6474	1.618	19	1.4664	3.666
10	0.7554	1.888	20	1.5950	3.988

angles, allowing measures of diameter in both co-ordinates with the same setting of the plate. The magnifying power used was 23.

PATROL CAMERA

In April 1911, the Observatory received from Brashear a doublet lens of 5 cm aperture and 40 cm focal length, with curves calculated by Hastings especially to give smooth extra-focal images. This was pointed toward the pole in a stationary box, and plates were exposed simultaneously with the zone plates, giving extra-focal trails 1 mm wide. Any change in the transparency of the air during the exposure would be registered on the trails and easily detected if exceeding 0.02 or 0.03 magnitudes. After the use

FIG. 5.—Patrol plate showing two polar trails

of this camera (which perhaps should be called a "sentinel" rather than a "patrol") was begun, one night which seemed uniform to the eye was proved to be changing by the trails, and the zone plates were rejected. Fig. 5 is a specimen patrol plate.

PROGRAM OF EXPOSURES

The useful field of the camera being about 6° in diameter, the program was arranged so that each star should appear on at least two plates. The setting in declination and the interval in right ascension were as follows:

Dec.	Interval	Dec.	Interval
75°	40^m	85°	4^h
79	1^h	87	4
83	2	90	8

As the space in this *Journal* does not permit the publication of the separate results for magnitude, there is no advantage in giving the journal of plates, further than to say that they were taken between 1908 May 22 and 1911 July 14. The number of plates used in the reductions is as follows:

Extra-focal for photographic magnitudes	122
Focal, for "visual" magnitudes	113
Objective-prism, for spectra	90
Total	325

PLATES, EXPOSURE, AND DEVELOPMENT

The plates used were 4×5 inches (10.1×12.6 cm) in size. For the photographic magnitudes, Seed "*Gilt Edge*" No. 27 (six No. 30 in 1911), and for the "visual" magnitudes, Cramer *Trichromatic* plates were used. An exposure of 30 minutes in the camera was usually sufficient to give images of the faintest stars needed. The exposures of the objective-prism plates ranged between 20 and 30 minutes. The plates were developed with hydroquinone for ten minutes (see exceptions noted below) in a metal tray floating in a tank of water which was kept at a temperature of $+20°$ Centigrade. The development was carried on in the dark.

DETERMINATION OF PHOTOGRAPHIC MAGNITUDES

The photographic magnitudes were obtained from the images taken 6 mm inside the focus of the camera, on Seed 27 plates. The opacity of the images was measured with the Hartmann "Mikrophotometer," the measures being reduced to magnitudes by the aid of the "absolute scale" obtained from the sensitometer images impressed on the plate. Great care in securing uniformity in handling the plate, left two sources of error which were under only partial control: (1) sky-fog, which darkened the film, and affected the reduction curve for the fainter stars; and (2) local differences in the sensitiveness of the film. Fig. 6 shows specimen reduction curves for measures of plates in the "Mikrophotometer." The readings on the millimeter scale of the instrument are ordinates and differences in magnitude (from sensitometer images) abscissas.

The scale-readings increase from faint to dense images; the magnitudes increase in the usual manner from bright to faint stars. The solid curve corresponds to a clear film on the negative; but any sky-fog acts as a supplementary exposure, and makes an appreciable increase in the opacity of the fainter images, thus raising the lower end of the curve. The dotted lines show this effect for film readings 10 and 12 on the scale. It is evident that each plate must be reduced with a curve corresponding to the reading

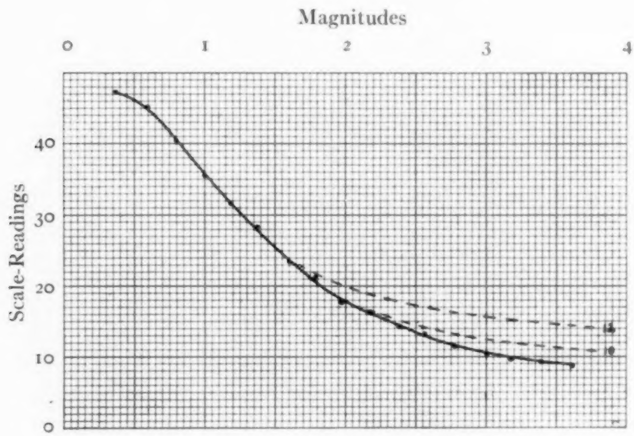


FIG. 6.—Reduction curves for extra-focal plates

of its film. A check on the determination of this curve was made by putting a second exposure on the plate, giving pairs of star-images which must differ by the same number of magnitudes. Usually the exposure times were 25 and 5 minutes, respectively. The denser images would fall on the part of the curve unaffected by the sky-fog; and the fainter image of the pair, separated from the other by a constant difference in magnitude, would serve to locate the lower part of the reduction curve.

As a check on the uniformity of sensitiveness of the plate, photometer readings were taken on the unexposed film near each star-image.

Correction to the center of the plate.—Images of *Polaris*, with constant exposure times, were taken at a series of positions on the

plate, and from measures of their opacities a correction curve was deduced to reduce each image to the center of the plate. It was found that the correction, expressed in magnitudes, was the same for star-images of different densities. The amounts of the corrections were—

Distance from Center	Correction	Distance from Center	Correction
0°0	$+0^{M}.00$	2°0	$+0^{M}.14$
0.5	+0.01	2.5	+0.22
1.0	+0.03	3.0	+0.32
1.5	+0.08		

Correction for atmospheric absorption.—The values given by Wirz were used for the photographic magnitudes. These values were determined from extra-focal plates reduced with the absolute scale.[1] As in the preceding paragraph, the corrections were applied to reduce to the center of the plate. As the zenith distances were usually less than 50° the amount of this correction was seldom greater than 0.02 magnitudes.

Standard stars.—Before the reduction of the photometric plates the spectral types of the stars were estimated on the Harvard classification from the objective-prism plates. Those stars were selected for standards which were as nearly white as Class A5 F. To the Potsdam visual magnitude was added a correction amounting to 0.04 for each step from A toward F, and the corrected quantity taken as the photographic standard magnitude. This provisional correction was confirmed by the final results of the zone work.

A few fields contained only two or three standard stars, but the number averaged six per plate. The mean of the corrected curve readings for the standard stars, subtracted from the mean of the photographic magnitudes, gives M_0, the quantity to be added to each curve reading to give the magnitude.

Systematic corrections.—The methods outlined permit the construction of a provisional catalogue, and a comparison of the separate stars and plates with the means from this catalogue gives data for a number of systematic corrections.

[1] *Astronomische Nachrichten*, **154**, 349, 1900; *Astrophysical Journal*, **31**, 23, 1910.

1) Correction to the reduction to the center. For stars having a distance from the center greater than 2°5, the following improvements to the reduction curve were found:

Dist.	Cor.	Dist.	Cor.
2°6	$-0^{M}.01$	3°2	$0^{M}.00$
2.8	−0.02	3.3	−0.04
3.0	−0.02	3.4	−0.07

This correction was applied to all the extra-focal plates.

2) Correction for scale and zero point. After the corrections for reduction to the center had been applied, the residuals for each plate were arranged in order of magnitude. (*a*) The slope of this curve showed the scale-error of the plate, as compared with the mean of all overlapping plates. (*b*) The distance of the curve, from the axis at a point corresponding to the mean magnitude of the standard stars used on the plate, showed the error of those standards as compared with the standards on all the overlapping plates. The application of these corrections to the individual stars therefore corrects for scale-error of the plate, and also refers the stars to the mean of the standards on all the overlapping plates, and in a less degree to the mean of all the standard stars in the entire zone.

Correction for color-error of the objective.—The color-curve of the Zeiss lens is flat in the region of the photographic rays, but in the visual part of the spectrum the curve is quite steep, the focal length for the wave-length of maximum visual sensitiveness being 2 mm longer than for the photographic maximum. As the plates were taken 6 mm inside the focus for the photographic rays, the light of the yellow stars would be even more out of focus, and their images would be larger and therefore less dense. The correction to be applied for this color-effect was determined in two ways: (1) by measuring the diameters of the images and calculating the change in magnitude from the change in area; (2) by making two exposures on the same plate, at equal distances inside and outside the focus, so that the color-effect would have the opposite sign. From the accordant results of these two methods the following table of corrections was found:

Uncorrected Color Index	Correction	Uncorrected Color-Index	Correction
0.25	$-0^{M}.01$	1.25	$-0^{M}.07$
0.50	−0.02	1.50	−0.09
0.75	−0.03	1.75	−0.11
1.00	−0.05	2.00	−0.14

This correction is the same in sign but only about one-fifth the amount of that found by Hertzsprung[1] for the Zeiss "UV" triplet at Potsdam.

Weights in the final catalogue.—The final weights were assigned from the following table, based on the progressive loss in uniformity of the images at increasing distances from the center.

Distance from Center	Weight	Distance from Center	Weight
$0^{\circ}.0$ to $1^{\circ}.5$	6	$2^{\circ}.5$ to $2^{\circ}.7$	3
1.6 to 2.0	5	2.8 to 3.0	2
2.1 to 2.4	4	3.1 to 3.4	1

A search was made for systematic differences in the magnitude results from the preceding and following sides of the plate (but with negative results). For the 75° zone the mean residuals were—

	Preceding Half	Following Half
Tel. East..............	−0.004	+0.003
Tel. West..............	−0.007	+0.001

If these errors were the result of faulty collimation, the general mean would be found by combining the preceding half Tel. East with the following half Tel. West, and vice versa. Weighted according to the number of plates these combined residuals become–

Preceding half.......................	$-0^{M}.003$
Following half.......................	0.000

The systematic differences were therefore negligible.

DETERMINATION OF "VISUAL" MAGNITUDES

Experiments made by Jordan and the writer, with the 24-inch reflecting telescope, demonstrated the possibility of obtaining magnitudes on the visual scale with a color-filter and color-sensitive

[1] *Astronomische Nachrichten*, **186**, 180, 1910.

plates.[1] A filter 4×5 inches in size, and identical in composition with that used on the reflector, was made by Mr. R. J. Wallace in February 1907. The spectral intensity curve of this filter with Cramer Trichromatic plates is shown in Fig. 7. A comparison of this curve, with those given by Ives[2] for the sensibility of the eye to lights of high and low intensity, will show that the sensibility curve of the plate falls between the high and low intensity curves

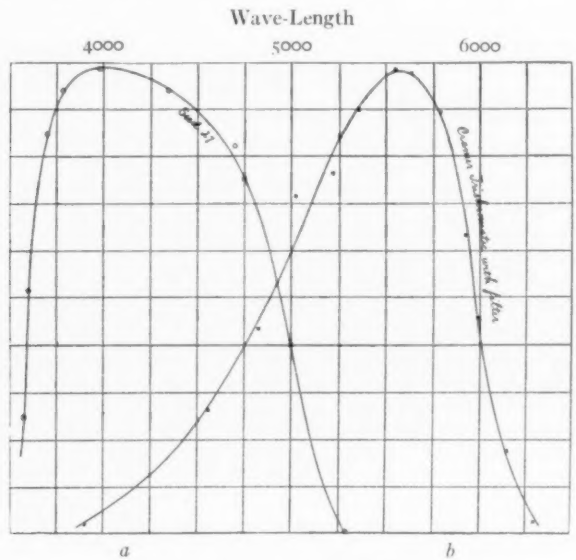


FIG. 7.—Spectral intensity curves

a, Seed 27, without filter
b, Cramer Trichromatic with "visual-luminosity" filter

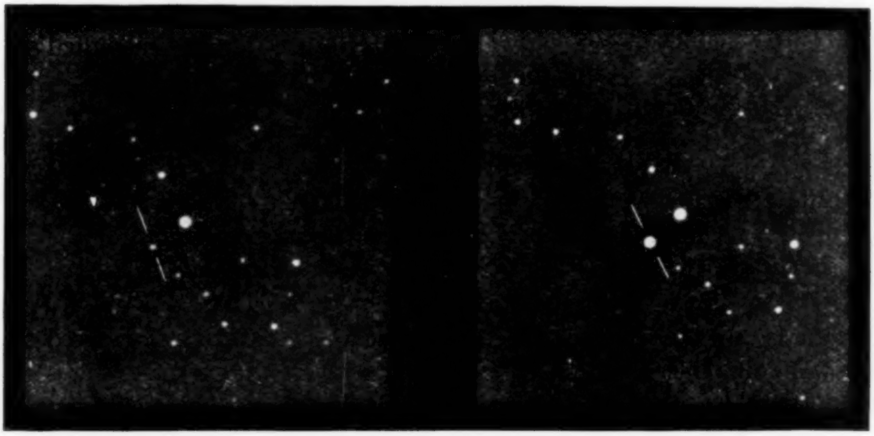
for the eye. Fig. 8 shows in a striking manner the capacity of the plate and filter to reproduce visual magnitudes of colored stars. The red star *U Cygni* (color-index 5.6 magnitudes), located by the short lines, is shown faint on the photographic (Seed) plate, but has its true visual magnitude on the filtered plate.

Exposure and development.—As exposures of four or five hours would be required to take extra-focal plates with the filter, it was

[1] "The Photographic Determination of Star-Colors and Their Relation to Spectral Type," *Astrophysical Journal*, **27**, 169, 1908.

[2] P. G. Nutting, *Outlines of Applied Optics*, p. 122.

decided to take these plates in focus, at the risk of some loss in accuracy of measurement. However, the loss was less than expected, on account of the remarkable sharpness of the star-images, in spite of the fact that the portion of the spectrum passed by the filter lies in the steep part of the color-curve. The diameters of these images could be measured with considerable precision under the microscope; also these diameters were less sensitive to



a *b*

FIG. 8.—The red star *U Cygni*

a, On Seed 27 plate, without filter

b, On Cramer Trichromatic plate with "visual-luminosity" filter

conditions of development than were the opacities of the extra-focal images; so that measures on different plates were nearly as consistent as those obtained by the extra-focal method. Previous to January 1910, the Cramer plates were developed in precisely the same manner as the Seed plates;[1] but unless the Cramer plates

[1] DEVELOPER FOR SEED PLATES

A		B	
Water	64 oz.	Water	64 oz.
Sodium sulphite, dry	2 oz.	Sodium sulphite, dry	2 oz.
Hydroquinone	2.5 oz.	Potassium carbonate	8 oz.
Sulphuric acid, c.p.	2 dr.	Potassium bromide	200 gr.

Equal parts A and B

were less than three months old there was a tendency to fog with this treatment, so the remainder were developed with "Cramer Extreme Contrast"[1] developer, for six minutes, which gave identical values of the reduction constants.

Reduction to the absolute scale.—As the method by sensitometer images could not be applied to plates taken in focus, the reduction constants for a mean plate were obtained by reducing Pleiades[2] exposures with values of the magnitudes found by the absolute scale. As already mentioned, this scale was confirmed later by the "Halbgitter" method, and the deviations of the individual plates were eliminated by systematic corrections.

It was found by trial that the reduction formula

$$\text{Magnitude} = a - b\sqrt{D}$$

fitted the camera and plates within the accidental errors of measurement. If the logarithm of the diameter were substituted for the square root, the representation is much worse. In Fig. 9 the same values of magnitude and diameter of the Pleiades stars are used, but in the upper part the square roots of D are platted while in the lower part the logarithms are used. Exposures ranging from 3 to 1000 seconds agreed in showing that the reduction "curve" was a straight line. The value of b, the tangent of

[1] DEVELOPER FOR CRAMER PLATES

A		B	
Water	64 oz.	Water	64 oz.
Sodium sulphite, dry	2 oz.	Sodium carbonate, dry	2 oz.
Hydroquinone	3 oz.	Sodium sulphite, dry	6 oz.
Sulphuric acid, c.p	2 dr.	Potassium carbonate	6 oz.
		Potassium bromide	240 gr.

Equal parts A and B

[2] Following are the magnitudes of the Pleiades stars found from extra-focal camera plates. The notation is Bessel's, and the magnitudes are expressed on the "International System."

Star, Magnitude	Star, Magnitude	Star, Magnitude
a 3.04	*g* 5.40	24 6.79
f 3.75	28 5.51	29 6.96
b 3.75	*m* 5.63	40 6.98
c 3.92	34 6.18	19 6.93
d 4.24	*s* 6.77	22 7.10
e 4.29	38 6.77	10 7.26
h 5.06	12 6.73	39 7.36

the inclination of this line, ranged from 0.50 to 0.53 for the different individuals taking part in the measurements.

Sources of error.—The principal sources were two: (1) error in focusing the camera, causing errors in the reduction to the center of the plate; (2) unsteadiness in the air, making a change in the value of b in the formula.

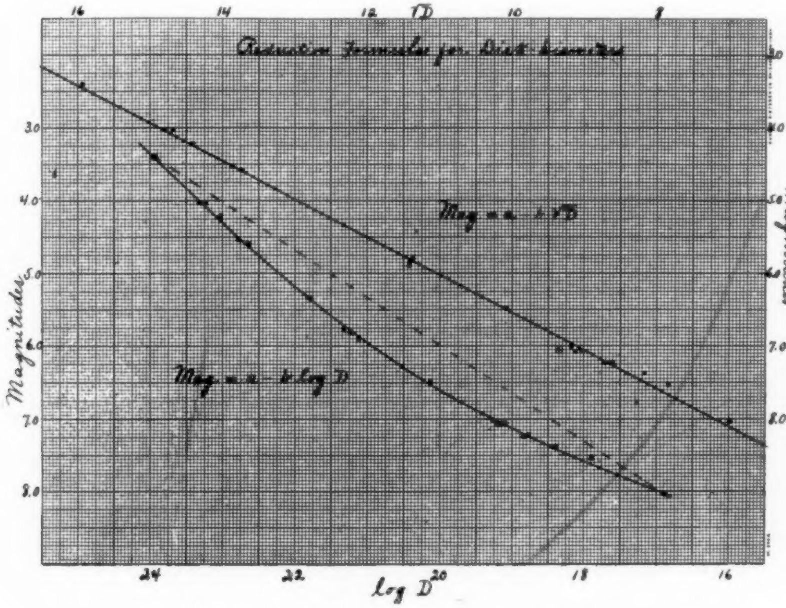


FIG. 9.—Reduction formulae for focal plates

Correction to the center of the plate.—This was determined in the same manner as the extra-focal corrections described on p. 179. The values were as follows:

Distance from Center	Correction	Distance from Center	Correction
0°0	0.M00	2°0	+0.M10
0.5	+0.02	2.5	+0.07
1.0	+0.07	3.0	0.00
1.5	+0.10		

No images at a distance greater than 3°0 were used. The above table applies directly to images for which the value of $b\sqrt{D} = 5.00$.

For any image the correction is represented within the accidental errors by the correction just given, multiplied by the factor

$$1+0.82(b1\ D-5.00).$$

Correction for atmospheric absorption.—The values used were those given in Vol. **3** of the Potsdam publications, p. 285. They were applied to reduce to the center of the plate.

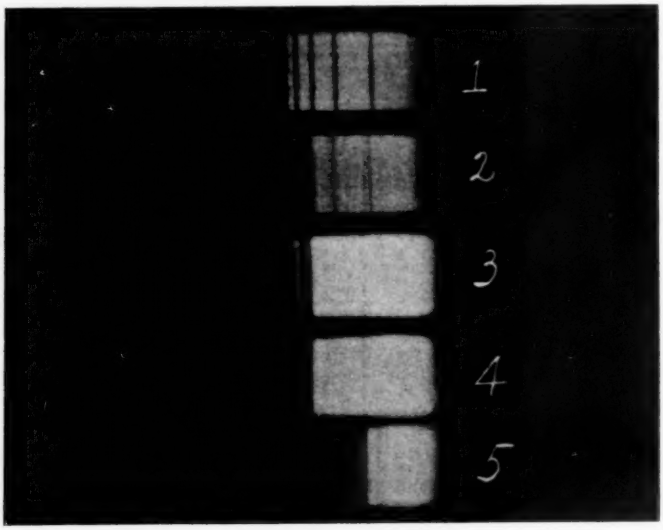


FIG. 10.—Classes of objective-prism spectra

1. *β Persei*, class B8
2. *α Persei*, class F5
3. *α Aurigæ*, class G0
4. *Saturn*, class G0
5. *α Tauri*, class K5

Standard stars.—These were chosen as for the photographic magnitudes, only in this case the Potsdam magnitudes were used as given in Vol. **17**. The corrected value for $b1\ D$ for each standard star gave a value of the constant a, and the mean for all the standards gave the value of a for the plate.

Systematic corrections.—These were obtained in the same manner as described for the extra-focal plates on p. 180. The corrections to the reductions to the center for distances greater than 2°5 were as follows:

Distance from Center	Correction	Distance from Center	Correction
2°6	$-0^{M}.02$	2°9	$-0^{M}.11$
2.7	-0.07	3.0	-0.13
2.8	-0.10		

Weights.—Stars not farther from the center of the plate than 2°5 were given weight 2, those farther out were given weight 1.

SPECTRAL CLASSIFICATION

The objective prism of 15° refracting angle, fitting over the cell of the doublet, gives spectra with a scale of 3.0 mm from $H\beta$ to $H\theta$. The field is flat in the region of the photographic rays, but the focal length increases rapidly for the visual region, so that only the region of shorter wave-length than λ 5000 could be used in the classification. Between this point and λ 3500 the focus is sharp.

The spectra were broadened by the diurnal motion. The clock was left running, and by using the lost motion in the right ascension screw, the star was allowed to trail to a width of about 0.5 mm a number of times sufficient to give a faint image of a star of magnitude 8.5 photographic. If there were stars brighter than 7.0 in the field, a second exposure with a less number of trails was also given.

In the effort to approximate the classification as closely as possible to the Harvard system, spectra of the following typical stars were taken:

Class	Star	Class	Star
Bo	*ε Orionis*		*π Tauri*
Ao	*Vega*	Ko	*Arcturus*
A5	*Altair*		*δ₁ Tauri*
Fo	*δ Aquilae*		*ε Tauri*
F5	*Procyon*		1 *Hev. Draconis*
Go	*Capella*		*τ Draconis*
	η Pegasi	K5	*Aldebaran*
	η Aquilae		*β Ursae Minoris*
	10 *Camelopardalis*	M	*α Orionis*
G5	*γ Delphini*	N	19 *Piscium*
	ε Ursae Minoris		249a *Schjellerup*

In January 1911 Mrs. W. P. Fleming very kindly classified for me the spectra of 138 stars on three of the plates used in this work,

Nos. OP 374, 377, and 410. Her favorable opinion of the quality of the plates,[1] in spite of the small scale, was given in her *Report to the Committee on the Classification of Stellar Spectra of the Solar Union.*

By constant reference to these standard spectra, taken under similar conditions, the classification was made as closely as possible on the Harvard system. There remains, however, one source of uncertainty as to what is meant by the "Harvard System." In *Harvard Circular No. 172*, p. 1, spectra like that of the sun are called Class G5, while in the previous volumes, for example **28**, pp. 35 and 158, they are called Class G. In this work the earlier classification is adopted.

The systematic difference between Mrs. Fleming's classification and my own is given in the following table, expressed in units which are tenths of the difference between Classes A and F, etc.

Mean Class	No. Stars	Diff. F−P
A5	31	+0.8
F5	12	+1.4
G5	14	+3.1
K5	9	−1.0

On the 90 objective-prism plates used, 1674 spectra of 654 stars were classified. The spectra of 134 stars were marked doubtful, leaving 520 stars available for a study of the relation between spectra and color-index.

REDUCTION TO "INTERNATIONAL SYSTEM"

The systematic difference between the magnitudes given in *Potsdam*, **17** and *Harvard*, **45** was found for the 124 stars in the present work, having spectra between Classes B8 and A2 inclusive. The result was:

124 stars . . . . Potsdam-Harvard = $+0^{M}.265$.

At Professor Schwarzschild's suggestion the same differences were taken out for the stars within the same spectral limits included in his "*Katalog der Polsterne*,"[2] with the result:

40 stars . . . . Potsdam-Harvard = $+0^{M}.268$.

[1] *Astrophysical Journal*, **33**, 273, 1911.

[2] *Aktinometrie*, B, p. 42.

Schwarzschild's value[1] for this difference was $+0^{M}.29$, so it was thought best to adopt the mean between his value and mine. The catalogue values are therefore:

Potsdam $-0^{M}.28$.

These give the magnitude at the zenith, since the Potsdam values, used as the basis, were so expressed.

THE CATALOGUE

Column 1 gives the B.D. number of the star. This is in italics where the precession between 1855 and 1900 has changed the declination to the adjacent degree. An asterisk before the number indicates that the star is not in the Potsdam *Durchmusterung*, an "s" that it was used as a standard for magnitude.

Columns 2 and 3 give the position for 1900, taken from the Potsdam catalogue. The degree of declination, for column 3, is to be taken from column 1.

Column 4 gives the photographic magnitude on the "International System," based on the visual magnitudes of the white stars in the Potsdam *Durchmusterung*.

Columns 5 and 6 give the probable error of the catalogue magnitude and the number of plates.

Columns 7, 8, and 9 give the corresponding values for the "Visual" magnitudes, found from color-sensitive plates used with a "visual-luminosity" filter.

Column 10 gives the "Color-Index" = Photographic Magnitude minus Visual Magnitude. This is, in general, negative for stars of Class B, and positive for stars of Classes A to M.

Column 11 gives the spectral class, estimated from the objective-prism plates on the Harvard system.

Column 12, under "S−Pa" gives the differences between Schwarzschild's photographic magnitudes of polar stars (*Aktinometrie*, B, p. 42) and column 4.

Column 13 gives the differences between Müller and Kempf's visual magnitudes in *Potsdam*, **17** and column 7.

[1] *Aktinometrie*, B, p. 13.

CATALOGUE OF ZONE +73° TO +90°

B.D.		Place 1900 R.A.			Place 1900 Dec.	Photographic Mag.	Photographic p.e.	Photographic No. pl.	Visual Mag.	Visual p.e.	Visual No. pl.	Color-Index	Spectrum	Differences S−Pa	Differences MK−Pa
		h	m	s								M		M	M
s73°	2	0	3	23	39′.4	7.33	.01	4	7.30	.05	3	+ .03	A1	......	+ .02
s78	1		3	47	9.6	6.09	.02	5	5.97	.05	4	+ .12	A3	......	+ .09
73	4		5	55	55.7	8.16	.04	2	7.05	.03	3	+1.11	G±	......	− .07
75	4		8	29	28.3	8.12	.04	2	7.21	.02	3	+ .91	G±	......	− .21
s76	5		10	33	23.7	6.21	.03	7	6.46	.02	3	− .25	A0	......	− .15
s75	7	0	12	43	43.3	6.99	.01	3	7.15	.04	3	− .16	A0	......	− .03
s79	10		20	42	29.9	6.20	.02	4	6.40	.05	4	− .20	B8	......	+ .08
76	10		24	28	28.1	7.05	.04	5	6.18	.04	2	+ .87	G4	......	− .02
81	13		32	13	56.5	6.87	.02	10	6.25	.03	5	+ .62	F5	− .01	+ .07
76	14		33	23	19.3	7.85	.03	5	6.64	.08	3	+1.21	K0	......	− .18
*75	36	0	38	10	23.6	7.84	...	1	7.12	.03	2	+ .72	G0	......	......
s74	27		39	2	26.5	5.88	.04	5	5.67	.05	3	+ .21	A1	......	+ .03
s74	29		41	5	18.1	5.39	.06	5	5.51	.03	3	− .12	A0	......	− .09
77	25		41	33	55.3	8.05	.02	3	6.91	.06	4	+1.14	G3	......	+ .04
77	27		43	41	24.5	6.90	.05	8	6.59	.03	5	+ .31	A7	......	− .05
72	43	0	44	19	1.4	8.58	.04	2	7.69	.06	2	+ .89	G±	......	− .07
s82	20		45	30	9.9	5.62	.02	6	5.50	.03	4	+ .12	A1	− .02	+ .06
73	40		46	30	46.2	7.75	.06	2	7.55	.10	3	+ .20	A5	......	− .08
79	24		52	8	0.3	6.98	.02	7	6.64	.03	5	+ .34	F0	+ .04	− .06
s83	20		52	49	4.1	6.87	.01	8	6.66	.07	4	+ .21	A4	− .01	+ .04
85	19	0	55	1	43.2	5.47	.01	8	4.25	.03	13	+1.22	G4	+ .08	− .11
s88	4		55	37	29.3	6.43	.02	6	6.45	.02	10	− .02	A1	+ .01	− .03
s73	51		57	38	50.1	6.75	.04	3	6.73	.03	2	+ .02	A1	......	− .01
86	17		59	5	36.8	7.35	.02	6	6.14	.03	9	+1.21	G5	− .10	− .12

79	29	I	0	40	28.7	7.16	.03	5	6.17	.06	4	+ .99	G2	......	− .06
s78	34		3	36	8.5	5.32	.03	4	5.59	.04	4	− .27	B9	......	− .09
73	59		6	1	23.5	8.27	.00	2	7.58	.05	2	+ .69	G±	......	− .03
79	36		7	40	22.7	6.59	.01	4	6.18	.01	4	+ .41	F1	......	.00
*73	60		9	2	28.9	....	...	..	7.86	.02	2	......	..	......	......
73	61		9	6	29.9	6.97	.07	4	6.96	.04	2	+ .01	A±	......	+ .06
80	35	I	9	41	20.0	7.55	.06	4	7.14	.03	5	+ .41	Fo	− .01	− .14
s80	36		10	3	22.0	6.63	.02	6	6.58	.03	5	+ .05	A1	− .05	+ .04
76	39		11	6	16.2	8.37	.03	3	7.12	.00	2	+1.25	G–K	......	− .04
76	40		11	59	2.5	7.23	.01	6	6.23	.04	5	+1.00	G4	......	− .08
78	30	I	13	9	30.1	8.57	.16	2	7.06	.05	3	+1.51	K5	......	− .12
73	66		13	46	3.4	7.29	.05	4	7.12	.04	2	+ .17	A4	......	− .04
75	58		13	48	11.2	7.89	.03	3	7.12	.05	3	+ .77	G2	......	− .04
75	59		13	51	42.9	6.62	.02	4	6.49	.07	3	+ .13	A5	......	− .19
77	49		14	57	12.2	6.31	.01	6	5.84	.02	3	+ .47	B5?	......	+ .11
73	75	I	21	0	41.4	7.74	.03	3	7.15	.04	3	+ .59	F8	......	− .05
*73	76		21	28	42.9	....	...	..	9.00	...	..	......	..	......	......
88	8		22	33	46.4	2.53	...	..	2.11	...	..	+ .42	..	......	......
s73	81		29	9	47.4	6.53	.04	4	6.62	.04	2	− .09	Ao	......	− .14
s80	50		29	38	55.2	7.01	.01	6	7.01	.03	5	+ .00	Ao	− .05	+ .01
s77	58	I	31	35	27.6	6.64	.01	7	6.64	.06	4	.00	Ao	......	+ .04
73	92		35	19	6.0	7.52	.00	2	6.55	.06	2	+ .97	G3	......	− .13
s75	72		35	57	22.1	7.37	.03	3	7.18	01	2	+ .19	A4	......	− .02
s80	55		38	50	23.2	6.94	.01	6	6.86	.04	4	+ .08	Ao	+ .14	+ .10
s80	57		39	46	52.6	7.52	.03	4	7.35	.03	4	+ .17	Ao	− .07	.00
74	84	I	42	48	5.6	7.25	.00	3	6.86	.04	2	+ .39	F4	......	− .08
81	61		43	31	27.9	8.59	.08	4	6.70	.04	4	+1.89	K5	......	− .05
80	58		44	35	25.0	8.48	.06	3	7.10	.06	5	+1.38	Ko	− .37	.00
77	65		44	44	42.2	7.75	.04	4	6.47	.08	4	+1.28	Ko	......	+ .01
s75	76		46	12	43.9	7.20	.02	3	6.88	.01	2	+ .32	A5	......	.00

CATALOGUE OF ZONE +73° TO +90°—*Continued*

B.D.		Place 1900 R.A.			Place 1900 Dec.	Photographic Mag.	Photographic p.e.	Photographic No. pl.	Visual Mag.	Visual p.e.	Visual No. pl.	Color-Index	Spectrum	Differences S−Pa	Differences MK−Pa
		h	m	s								M		M	M
74°	87	1	48	48	50.′8	7.52	.02	2	6.59	.03	2	+ .93	G2	......	− .13
75	83		51	17	28.0	8.62	.08	2	6.96	.04	3	+1.66	K±	......	− .22
77	73		52	49	25.9	7.23	.03	6	5.92	.09	3	+1.31	G9	......	− .14
s74	*91*		52	52	1.1	6.48	.02	5	6.59	.01	3	− .11	A0	......	− .07
s73	108		54	19	22.0	6.32	.01	3	6.11	.04	2	+ .21	A3	......	+ .05
76	63	1	55	5	48.1	5.58	.05	5	5.52	.04	3	+ .06	A5	......	− .38
s79	*61*		55	43	11.1	7.64	.03	4	7.51	.05	4	+ .13	A2	− .15	− .17
75	86		55	58	38.1	6.24	.00	3	5.45	.05	3	+ .79	G2	......	− .43
s73	*112*		56	55	6.2	7.53	.07	2	7.40	.02	3	+ .13	A1	......	− .05
s80	64		57	4	49.1	6.03	.01	6	5.95	.05	5	+ .08	A2	.00	+ .05
s74	95	1	57	53	23.5	7.59	.09	2	7.58	.01	3	+ .01	A2	......	+ .08
78	69		57	53	52.1	7.32	.03	5	6.98	.06	4	+ .34	A6	......	− .08
s80	*65*		57	53	0.3	6.96	.02	6	6.77	.05	5	+ .19	A4	+ .02	− .11
78	*71*	2	1	6	13.1	6.20	.02	8	7.19	.04	4	......	A2	......	− .29
s79	63		1	25	13.1				6.45	.03	4			......	− .05
82	*51*		1	25	5.5	7.28	.03	4	6.36	.06	3	+ .92	G2	......	− .12
73	121		4	8	33.4	7.16	.03	3	6.18	.04	3	+ .98	G3	......	− .06
s78	73		4	41	42.7	7.06	.03	4	6.99	.08	3	+ .07	A1	+ .31	+ .19
s75	94	2	14	39	40.6	7.60	.06	3	7.55	.03	2	+ .05	A0	......	+ .09
72	*128*		15	19	2.6	8.19	.09	3	8.11	.04	2	+ .08	A±	......	+ .13
73	139		18	22	15.7	7.54	.03	3	7.18	.02	2	+ .36	A7	......	− .14
77	84		22	31	13.1	8.33	.04	4	7.68	.04	3	+ .65	G±	......	− .06
79	75		23	41	17.3	8.45	.04	3	7.40	.07	2	+1.05	G2	......	− .02

76	81	2	23	44	16.6	8.55	.04	4	6.91	.01	2	+1.64	K±	......	− .35
74	*111*		27	48	5.5	8.14	.06	3	7.94	.04	2	+ .20	A–F±	......	− .04
74	112		27	51	56.7	8.46	...	1	8.20	.06	2	+ .26	......	......	− .04
80	*86*		33	21	1.5	7.01	.05	6	5.67	.02	3	+1.34	K2p	+ .15	− .09
s74	117		36	6	59.1	7.30	.01	3	7.13	.01	3	+ .17	A3	......	− .01
s79	86	2	41	47	41.6	7.27	.03	6	7.25	.01	2	+ .02	A2	......	− .17
s74	120		42	12	19.5	7.74	.01	4	7.57	.02	3	+ .17	A4	......	+ .13
75	*109*		44	15	7.1	8.57	.06	3	7.42	.03	3	+1.15	G5±	......	− .12
76	101		47	18	40.5	7.78	.01	3	6.90	.04	3	+ .88	G2	......	− .21
78	*103*		52	47	1.4	7.25	.04	6	5.38	.03	3	+1.87	M	......	− .18
s74	131	2	53	20	45.2	6.93	.02	3	7.08	.03	2	− .15	B8	......	− .04
s73	165		54	52	33.1	7.44	.02	3	7.34	.01	2	+ .10	A2	......	+ .12
s*80*	*97*		56	11	5.0	5.87	.02	8	5.71	.02	5	+ .16	A4	+ .13	− .01
75	124		56	16	24.8	8.51	.06	2	7.07	.01	2	+1.44	K±	......	− .28
75	127		58	28	41.3	7.83	.02	3	7.48	.01	2	+ .35	A–F	......	......
77	109	3	0	46	48.6	8.61	.03	2	7.13	.04	4	+1.48	K±	......	− .19
s*73*	*168*		1	5	0.9	4.46	.03	3	4.82	.01	2	− .36	A0	......	+ .02
79	94		1	27	45.2	7.68	.05	4	6.59	.06	3	+1.09	G4	......	+ .10?
s73	170		2	36	55.0	7.66	.05	3	7.04	.02	2	+ .62	F2	......	+ .08
73	171		3	26	51.9	8.39	...	1	6.74	.01	2	+1.65	G–K±	......	− .09
77	*111*	3	3	45	7.6	8.62	.02	2	7.33	.02	3	+1.29	K0	......	+ .39
s78	109		4	1	30.0	6.89	.02	4	6.74	.11	3	+ .15	A3	......	− .06
73	172		4	25	29.6	7.97	.03	3	6.45	.01	2	+1.52	K+	......	− .16
81	107		6	30	47.0	8.42	.06	3	6.82	.03	4	+1.60	K5	......	+ .03
s77	115		7	37	22.1	5.27	.07	6	5.45	.06	4	− .18	A3	......	− .10
82	*82*	3	8	7	10.1	8.58	.09	3	7.04	.07	3	+1.54	K5	......	.00
s74	144		8	34	52.2	7.24	.03	4	7.23	.03	3	+ .01	A1	......	+ .03
84	59		8	34	33.5	6.38	.02	8	5.45	.06	4	+ .93	G0	+ .16	+ .02
73	179		10	15	20.0	8.09	.03	4	6.66	.01	3	+1.43	G–K±	......	− .12
73	180		12	57	48.9	7.20	.04	4	6.74	.01	3	+ .46	F1	......	− .03

CATALOGUE OF ZONE +73° TO +90°—*Continued*

B.D.		Place 1900				Photographic			Visual			Color-Index	Spectrum	Differences	
		R.A.			Dec.	Mag.	p.e.	No. pl.	Mag.	p.e.	No. pl.			S−Pa	MK−Pa
		h	m	s								M		M	M
77°	*119*	3	14	38	7′.4	8.16	.03	2	7.47	.03	3	+ .69	F5	......	+ .14
*74	151		18	22	55.9	7.98	...	1	7.87	.07	3	+ .11	A3	......	......
77	123		18	31	39.9	8.58	.01	3	7.13	.06	3	+1.45	Ko	......	+ .06
s72	*178*		24	14	0.5	6.45	.08	3	6.40	.04	2	+ .05	A1	......	+ .12
74	161		27	1	24.2	8.24	.11	2	7.33	.01	3	+ .91	G2	......	+ .05
75	143	3	27	21	24.4	7.15	.01	4	6.10	.06	3	+1.05	G3	......	.00
76	128		29	41	50.6	7.87	.03	7	7.68	.06	2	+ .19	A1	......	+ .17
74	168		33	29	13.3	7.66	.05	4	6.57	.01	2	+1.09	G5	......	+ .03
s83	91		33	43	14.1	7.25	.02	6	7.58	.05	3	− .33	B7	+ .16	+ .06
86	51		33	54	20.0	6.13	.03	8	5.78	.07	4	+ .35	Fo	− .01	− .06
s77	133	3	34	56	48.3	6.88	.02	4	6.90	.06	4	− .02	Ao	......	+ .08
74	174		42	4	21.8	8.56	...	1	7.62	.04	2	+ .94	G±	......	+ .11
73	204		44	57	46.9	8.16	.06	2	6.58	.00	2	+1.58	Ko	......	− .16
77	138		49	2	55.0	7.95	.03	3	6.70	.06	4	+1.25	G5	......	+ .04
75	*160*		53	8	8.3	7.94	.03	4	7.61	.02	3	+ .33	A8	......	+ .09
80	125	3	53	17	25.4	5.44	.04	4	4.93	.04	4	+ .51	F4	+ 19	+ .11
*78	142		54	19	39.7	7.63	.03	4	....	...	..	......	Ao	......	......
*74	184		54	53	54.7	7.44	.01	2	7.01	.05	3	+ .43	F2	......	......
74	186		55	19	21.8	7.66	.03	2	6.56	.01	3	+1.10	G5	......	+ .08
73	210		55	22	43.6	7.40	.02	2	6.26	.05	3	+1.14	G5	......	+ .18
78	146	3	57	8	45.7	8.09	.02	2	6.74	.04	3	+1.35	G7	......	− .02
73	212		57	44	17.9	7.16	.03	2	6.50	.03	3	+ .66	F5	......	+ .12

80	127	4	1	4	16.6	7.35	.02	6	6.20	.02	4	+1.15	G4	− .04	+ .03
s81	147		1	58	43.3	7.32	.02	4	7.35	.01	2	− .03	A0	+ .04	+ .05
s83	104		4	59	33.9	5.04	.07	6	5.50	.01	3	− .46	B6	+ .26	+ .09
85	63		5	5	17.4	6.91	.02	8	6.41	.04	4	+ .50	F6	+ .05	+ .05
77	150		5	36	49.7	7.70	.04	5	6.66	.04	5	+1.04	G3	......	+ .18
82	*113*	4	7	59	6.0	6.19	.02	4	5.47	.10	3	+ .72	G0	+ .10	− .07
s75	173		8	12	51.6	6.56	.02	5	6.58	.02	4	− .02	A0	......	+ .12
84	78		8	58	14.5	8.42	.05	4	7.13	.03	5	+1.29	K0	......	+ .01
80	133		9	37	35.2	6.57	.01	6	5.28	.06	4	+1.29	K2p	+ .16	.00
s80	134		12	0	41.9	7.20	.01	6	7.10	.03	4	+ .10	A2	+ .06	− .03
80	140	4	19	9	39.8	7.49	.01	6	7.22	.03	4	+ .27	A5	− .04	+ .12
83	114		21	32	49.6	8.09	.04	3	7.04	.03	3	+1.05	G2	− .42	+ .12
78	157		22	9	46.5	7.48	.01	4	7.16	.02	3	+ .32	A6	......	+ .04
s79	150		28	50	27.6	6.56	.01	6	6.51	.03	4	+ .05	A1	......	+ .15
76	174		32	8	25.4	6.93	.02	8	6.31	.01	3	+ .62	F5	......	+ .06
s75	189	4	35	22	45.6	6.22	.02	5	5.90	.02	3	+ .32	A4	......	+ .02
75	193		39	31	32.4	7.77	.02	3	6.87	.07	3	+ .90	G1	......	+ .13
80	*155*		41	37	1.7	6.47	.01	8	5.06	.04	5	+1.41	K4	+ .03	− .06
77	178		46	1	37.0	8.20	.03	4	7.64	.06	4	+ .56	F5	......	+ .06
74	229		49	38	6.9	7.69	.02	3	5.94	.04	2	+1.75	K5	......	− .10
73	264	4	51	45	36.9	7.86	.03	3	6.06	.00	2	+1.80	K5+	......	− .04
s73	265		52	3	55.2	6.03	.01	5	6.07	.01	2	− .04	A0	......	+ .06
85	74		56	16	49.8	6.67	.02	10	6.37	.03	3	+ .30	A5	+ .03	+ .07
s75	208		56	29	32.8	7.19	.02	4	7.08	.06	3	+ .11	A2	......	+ .19
s73	274		59	45	49.1	5.13	.02	4	5.52	.03	3	− .39	B8	......	− .13
s76	190	5	0	30	20.8	6.27	.02	8	6.29	.00	3	− .02	A0	......	+ .05
s74	238		4	34	25.1	7.47	.02	4	7.27	.03	3	+ .20	A4	......	+ .05
s73	280		5	53	9.2	5.68	.02	4	5.82	.02	3	− .14	A0	......	− .04
79	169		6	5	6.9	5.41	.04	4	5.26	.04	2	+ .15	F5	......	− .32
75	220		8	41	5.4	8.21	.02	5	7.90	.03	3	+ .31	F0	......	+ .10

CATALOGUE OF ZONE +73° TO +90°—*Continued*

B.D.		Place 1900 R.A.			Place 1900 Dec.	Photographic Mag.	Photographic p.e.	Photographic No. pl.	Visual Mag.	Visual p.e.	Visual No. pl.	Color-Index	Spectrum	Differences S−Pa	Differences MK−Pa
		h	m	s								M		M	M
s85°	78	5	9	52	35′.3	6.53	.01	11	6.64	.03	5	− .11	A0	− .02	− .10
78	183		10	25	18.8	7.74	.02	3	6.74	.02	2	+1.00	G2	......	− .28
s83	141		11	48	46.6	7.01	.02	6	7.00	.04	4	+ .01	A0	+ .09	− .08
73	285		12	24	36.5	7.97	.06	4	6.45	.01	3	+1.52	K5	......	− .15
78	187		13	6	12.6	7.03	.03	4	6.75	.00	2	+ .28	F2	......	− .11
s77	195	5	14	2	53.1	6.64	.03	6	6.48	.02	3	+ .16	A3	......	− .02
s74	241		14	37	27.7	7.29	.02	6	7.19	.04	4	+ .10	A0	......	− .22
s74	242		15	35	13.0	6.88	.02	6	6.91	.03	4	− .03	A0	......	− .03
74	249		23	56	15.1	7.21	.02	6	6.98	.04	3	+ .23	A6	......	− .14
74	250		24	24	37.5	7.99	.03	5	7.56	.05	3	+ .43	F0	......	− .04
74	252	5	26	21	58.6	7.76	.02	5	6.22	.01	3	+1.54	K5	......	− .25
85	80		29	54	8.8	7.65	.04	8	6.07	.02	3	+1.58	K5	− .11	− .16
s73	298		30	11	55.7	6.91	.04	4	6.54	.11	3	+ .37	A9	......	+ .16
74	257		33	14	34.1	7.84	.03	4	7.25	.02	3	+ .59	F8	......	− .03
75	236		36	37	40.5	7.87	.04	4	7.54	.05	3	+ .33	F2	......	+ .02
75	247	5	51	22	34.9	7.68	.05	4	6.20	.06	4	+1.48	K5	......	− .11
73	311		52	24	59.9	7.76	.07	4	7.29	.01	2	+ .47	F2	......	− .01
77	229		57	58	18.0	8.21	.03	5	7.31	.03	4	+ .90	G1	......	− .05
76	226	6	0	38	31.5	8.07	.03	5	7.65	.01	3	+ .42	F0	......	− .01
79	198		4	15	48.8	8.93	.03	2	7.88	.03	5	+1.05	G±	......	− .09
76	234		4	58	51.6	8.33	v?	5	7.80	.04	3	+ .53	G±	......	− .22
86	79		8	3	45.6	7.69	.03	5	6.45	.01	3	+1.24	K1	− .10	+ .03
73	328		13	13	29.0	8.60	.02	2	7.55	.10	3	+1.05	G±	......	− .05

78	225	6	13	33	20.3	8.97	.01	2	7.90	.00	2	+1.07	G±	......	− .08
79	201		15	52	31.7	8.33	.02	3	8.05	.01	2	+ .28	Fo	......	− .10
79	202		16	33	2.4	7.86	.02	3	6.59	.01	2	+1.27	G8	......	− .14
80	210		16	57	38.2	8.54	.03	3	6.98	.04	3	+1.56	K5	......	+ .02
s78	226		17	41	13.8	7.01	.02	4	7.03	.02	2	− .02	B9	......	+ .07
76	244	6	20	58	56.2	8.44	.08	4	7.86	.01	2	+ .58	F–G	......	+ .22
s77	247		21	6	59.8	7.56	.03	5	7.49	.01	2	+ .07	A1	......	− .09
s79	208		23	6	40.6	6.74	.01	6	6.65	.03	2	+ .09	A1	......	+ .03
s82	177		23	23	12.0	6.68	.04	4	6.62	.06	3	+ .06	A3	− .01	− .06
78	227		24	57	4.5	7.25	.03	4	5.50	.01	2	+1.75	K5	......	+ .03
73	340	6	25	18	46.4	6.44	.03	3	6.24	.07	2	+ .20	Fo	......	− .10
80	217		29	3	20.0	8.51	.00	3	7.23	.03	3	+1.28	Ko	......	− .14
79	212		29	11	40.6	5.82	.03	6	5.39	.02	2	+ .43	F5	......	− .03
76	249		30	7	40.7	7.96	.04	5	7.00	.10	2	+ .96	G1	......	+ .03
78	234		38	44	7.4	8.12	.08	3	7.79	.05	2	+ .33	F1	......	− .21
75	273	6	45	5	19.3	8.07	.07	4	7.73	.04	3	+ .34	A8	......	+ .01
77	266		45	29	6.3	6.00	.03	6	4.62	.06	4	+1.38	K3	......	− .18
75	281		50	46	22.7†	7.22	.07	3	6.67	.04	2	+ .55	F5	......	......
75	280		50	48	33.0	7.57	.05	3	6.88	.08	2	+ .69	G2	......	+ .02
73	360		52	9	34.1	8.41	.02	2	7.39	.00	2	+1.02	G±	......	+ .01
87	51	6	53	45	12.3	6.70	.01	7	4.95	.06	9	+1.75	K5+	− .04	− .05
78	240		57	58	54.6	7.18	.01	4	6.81	.07	3	+ .37	A6	......	+ .13
73	366	7	3	46	29.4	7.99	.02	3	7.56	.01	2	+ .43	Fo	......	− .14
80	230		5	45	48.3	8.29	.04	4	7.07	.03	6	+1.22	G5	− .94?	− .03
s81	242		6	24	26.4	6.22	.02	8	6.25	.04	6	− .03	B8	− .12	+ .11
78	246		7	44	4.6	8.23	.03	2	7.67	.02	4	+ .56	F5	......	− .17
s75	296		8	47	16.3	6.85	.01	6	6.82	.01	2	+ .03	Ao	......	.00

† Σ 973, mean δ.

CATALOGUE OF ZONE $+73^\circ$ TO $+90^\circ$—*Continued*

B.D.		Place 1900 R.A.	Place 1900 Dec.	Photographic Mag.	Photographic p.e.	Photographic No. pl.	Visual Mag.	Visual p.e.	Visual No. pl.	Color-Index	Spectrum	Differences S−Pa	Differences MK−Pa
		h m s								M		M	M
82°	201	7 10 4	36′.3	6.62	.01	6	4.90	.02	4	+1.72	M	+ .04	− .12
78	250	10 8	26.5	8.61	.06	2	7.11	.04	4	+1.50	K2	......	− .11
78	251	11 7	14.1	7.58	.02	4	7.51	.03	4	+ .07	A2	......	− .05
s74	322	13 33	2.5	7.42	.01	4	7.49	.00	2	− .07	A0	......	− .15
*75	300	13 36	49	7.5	...	1	....	...	..	......	A0?	......	......
73	375	7 14 32	16.4	7.04	.01	4	7.14	.04	2	− .10	A–F	......	+ .08
*73	376	14 39	16.4‡	....	...	2	7.5	...	2	......	A±	......	......
81	252	16 27	6.0	7.62	.01	7	6.41	.05	4	+1.21	G5	− .18	− .01
77	286	17 3	9.3	8.32	.04	6	7.12	.05	4	+1.20	G+	......	+ .05
78	254	17 31	53.6	7.95	.01	2	7.49	.06	4	+ .46	F0	......	− .27
76	284	7 25 47	0.5	8.05	.05	6	7.60	.03	3	+ .45	F0	......	− .06
73	381	26 27	49.0	7.81	.02	3	7.38	.04	2	+ .43	F+	......	− .04
s*82*	*213*	27 41	54.8	7.75	.01	6	7.82	.02	4	− .07	A0	− .37	− .34
74	335	36 23	17.2	8.20	.05	3	6.96	.01	2	+1.24	G–K±	......	− .03
81	257	38 56	36.5	8.31	.14	2	6.92	.05	4	+1.39	K2	......	+ .01
80	238	7 39 46	31.0	7.31	.03	6	6.52	.03	4	+ .79	G1	− .19	− .07
80	240	43 20	7.3	8.36	.11	3	6.79	.04	4	+1.57	K4	......	− .19
s77	303	43 29	50.1	7.23	.02	4	6.80	.06	3	+ .43	A3	......	− .10
74	338	48 14	11.1	6.85	.02	6	5.48	.00	2	+1.37	K2	......	− .31
s79	265	49 5	45.2	5.41	.03	5	5.41	.04	3	+ .00	A0	......	− .11
86	*113*	7 52 28	59.4	8.12	.02	5	7.33	.03	3	+ .79	G0	− .25	− .13
s84	169	53 2	20.8	6.36	.01	11	6.36	.04	5	+ .00	A0	− .03	+ .04
78	*278*	57 6	55.9	8.41	.10	3	7.73	.04	3	+ .68	F±	......	+ .23
s*89*	*13*	58 3	56.0	7.12	.02	8	6.96	.03	10	+ .16	A3	+ .02	+ .04

76	308	8	4	14	2.8	7.77	.01	4	7.50	.04	3	+ .27	Fo	......	+ .12
s82	235		5	12	44.5	6.25	.06	3	6.30	.07	2	− .05	B9	− .12	− .08
76	310		6	59	3.7	6.39	.01	7	5.68	.03	3	+ .71	Go	......	− .23
75	334		10	8	7.8	7.24	.00	2	6.42	.01	3	+ .82	G2	......	− .14
78	284		10	45	15.2	8.42	.01	2	8.40	.10	3	+ .02	Ao	......	− .19
77	327	8	11	44	16.9	8.25	.07	4	6.75	.04	4	+1.50	K±	......	− .03
78	287		16	58	33.4	7.96	.05	2	7.06	.02	3	+ .90	Go	......	− .17
73	416		20	58	1.5	8.54	...	1	7.30	.01	2	+1.24	Ko	......	− .18
73	420		23	44	49.0	8.17	.07	2	7.24	.02	2	+ .93	Go	......	+ .04
77	337		24	20	23.1	7.70	.04	6	7.11	.02	5	+ .59	F5	......	+ .01
s75	342	8	25	11	3.9	6.43	.01	3	6.39	.04	3	+ .04	A2	......	− .15
85	128		25	25	24.6	7.62	.01	6	7.26	.03	4	+ .36	F3	− .09	+ .06
s82	253		28	19	35.6	6.74	.02	6	6.73	.03	3	+ .01	Ao	− .09	+ .11
74	*370*		28	36	58.8	7.07	.05	2	6.24	.02	3	+ .83±	G5	......	− .24
s73	428		31	59	31.4	6.78	.03	3	6.76	.04	3	+ .02	Ao	......	+ .08
78	291	8	35	2	3.1	7.86	.04	3	7.70	.04	4	+ .16	F5	......	− .18
s73	430		35	5	39.1	7.18	.00	2	7.19	.03	4	− .01	Ao	......	+ .19
78	293		37	31	31.9	8.54	.05	2	6.93	.03	4	+1.61	K5–M	......	− .19
74	379		40	5	7.3	8.40	...	1	7.16	.02	4	+1.24	K±	......	+ .08
s80	272		40	52	24.2	7.50	.04	7	7.62	.06	5	− .12	B9	− .28	− .06
83	232	8	41	48	5.9	8.11	.04	3	7.08	.05	3	+1.03	G3	− .29	− .11
83	233		44	31	7.6	7.30	.02	5	6.99	.10	3	+ .31	F1	− .10	− .13
78	297		45	23	31.5	7.61	.02	4	7.24	.02	4	+ .37	Fo	......	− .04
76	335		51	44	47.6	8.25	...	1	7.74	.02	3	+ .51	F±	......	− .09
s79	294		51	47	44.3	7.83	.01	7	7.70	.05	4	+ .13	A3	......	+ .10
84	196	8	54	32	35.0	6.47	.02	9	6.26	.05	3	+ .21	A6	.00	− .06
81	282		56	18	13.8	6.76	.03	6	6.27	.01	5	+ .49	F1	− .24	+ .03

‡ 32″ fol. 73°.375.

CATALOGUE OF ZONE +73° TO +90°—*Continued*

B.D.		Place 1900 R.A.			Place 1900 Dec.	Photographic Mag.	Photographic p.e.	Photographic No. pl.	Visual Mag.	Visual p.e.	Visual No. pl.	Color-Index	Spectrum	Differences S−Pa	Differences MK−Pa
		h	m	s								M		M	M
75°	365	9	2	51	41′.2	8.30	...	1	7.35	.04	3	+ .95	G±	......	+ .11
s79	300		3	59	41.5	7.89	.01	6	8.21	.02	3	− .32	B4	......	− .06
s77	359		5	17	29.2	7.01	.01	8	6.94	.03	6	+ .07	A0	......	+ .06
s73	452		5	51	21.6	6.08	.03	4	5.92	.01	4	+ .16	A2	......	+ .06
74	393		9	37	26.2	7.40	.03	2	6.30	.04	4	+1.10	G5	......	+ .08
s75	370	9	11	26	20.1	7.49	.01	3	7.21	.03	4	+ .28	A2	......	+ .14
79	*305*		14	13	51.8 }	8.33	.02	5	8.38	.02	2	......	F±	......	− .08
79	*305*		14	13	51.8 }				8.83	.02	2				− .23
s75	377		17	22	31.7	6.22	.02	6	6.16	.02	5	+ .06	A2	......	− .10
83	256		20	30	21.9	7.33	.02	5	6.91	.03	3	+ .42	F2p	− .10	− .01
81	302	9	22	51	46.1	5.83	.03	8	4.49	.03	5	+1.34	K2p	+ .09	− .35
s74	402		26	8	45.8	6.19	.01	5	6.43	.02	3	− .24	B9	......	− .10
s*83*	*262*		26	27	48.8	7.55	.01	4	7.49	.05	3	+ .06	A3	− .18	− .09
73	470		28	24	31.7	6.62	.01	5	6.29	.03	3	+ .33	A5	......	du
75	386		28	28	39.1	7.48	.06	3	6.58	.04	3	+ .90	G+	......	− .19
s73	471	9	30	5	11.7	7.15	.01	3	7.05	.02	3	+ .10	A3	......	+ .05
79	316		30	28	16.8	8.30	.03	4	8.28	.05	3	+ .02	A2	......	− .14
76	*360*		30	38	53.0	8.25	...	1	7.22	.03	3	+1.03	G+	......	+ .01
78	317		34	11	35.4	7.76	.01	5	6.20	.04	3	+1.56	K2	......	− .04
s75	389		34	39	2.6	6.86	.03	4	6.73	.02	3	+ .13	A5	......	+ .01
79	319	9	35	27	35.7	6.33	.01	6	6.14	.06	3	+ .19	A7	......	.00
78	327		46	5	24.6	8.50	.02	4	7.57	.04	3	+ .93	G±	......	− .17
81	*319*		46	52	51.2	8.56	.07	3	7.89	.05	4	+ .67	G±	......	− .04
77	389		47	44	5.2	8.48	.04	3	7.74	.04	6	+ .74	G±	......	− .01
73	478		49	27	21.3	6.99	.02	4	5.94	.04	3	+1.05	K2p	......	− .14

84	225	9	52	37	24.1	7.87	.04	6	6.13	.02	4	+1.74	K5	− .23	− .01
75	399		52	39	14.4	7.75	.09	3	6.84	.03	4	+ .91	G2	......	− .06
83	*280*		59	34	53.3	8.31	.07	2	7.26	.08	2	+1.05	G±	......	− .07
s79	328	10	5	49	26.5	6.95	.01	8	6.85	.03	4	+ .10	A2	......	+ .01
73	489		9	36	34.4	6.76	.03	5	6.51	.14	2	+ .25	A6	......	− .01
83	287		11	43	18.3	7.80	.03	2	6.73	.07	2	+1.07	G±	− .19	− .01
79	*330*		13	31	50.8	8.46	.03	3	7.22	.03	4	+1.24	G–K±	......	− .03
84	234		15	9	45.6	5.57	.02	10	5.39	.02	4	+ .18	A4	+ .07	+ .02
83	297	10	18	55	4.1	5.39	.02	6	5.07	.12	2	+ .32	A8	+ .16	+ .07
85	*161*		20	46	54.7	7.99	.01	7	7.15	.03	4	+ .84	G0	− .19	.00
78	349		22	53	0.6	8.89	.00	2	7.83	.04	3	+1.06	G±	......	+ .13
74	*436*		23	15	50.6	7.93	.04	2	6.90	.05	2	+1.03	G2	......	− .04
81	343		25	44	0.6	7.47	.02	6	6.19	.04	3	+1.28	K0	− .13	+ .23
s74	438	10	26	7	21.0	7.49	.03	3	7.36	.01	2	+ .13	A2	......	+ .02
76	393		26	36	13.7	5.87	.02	7	5.11	.09	4	+ .76	G±	......	− .39
74	440		33	15	17.7	7.82	.06	4	7.52	.04	3	+ .30	F2	......	+ .23
s*81*	*349*		33	38	56.9	6.71	.01	6	6.49	.03	3	+ .22	A1	− .16	+ .17
s*78*	*359*		34	31	56.0	7.57	.04	4	7.63	.04	3	− .06	A±	......	+ .09
76	402	10	43	28	31.5	7.27	.04	2	7.01	.07	2	+ .26	A4	......	+ .11
80	*338*		45	47	52.6	8.56	...	4	7.59	.09	2	+ .97	G±	......	− .04
77	412		47	11	37.2	8.40	.06	5	6.74	.06	3	+1.66	M±	......	− .14
*80	347		50	41	19.4	7.68	.06	4	....	...	..	......	F±	......	......
76	406		51	30	15.4	7.81	.05	4	7.42	.02	2	+ .39	A8	......	+ .08
78	367	10	51	58	18.4	7.20	.03	4	6.12	.04	2	+1.08	G3	......	− .06
79	348		53	14	25.3	8.38	.01	2	7.38	.09	3	+1.00	G±	......	.00
82	325	11	2	13	16.7	7.76	.03	7	7.10	.07	6	+ .66	F8	......	− .03
78	375		2	26	19.8	8.75	.08	2	6.92	.10	2	+1.83	K±	......	− .14
s86	161		2	33	11.0	7.36	.00	7	7.17	.04	4	+ .19	A5	− .05	− .03
s88	64		4	14	11.0	7.27	.02	10	7.45	.02	9	− .18	B9	+ .01	+ .01

CATALOGUE OF ZONE $+73^{\circ}$ TO $+90^{\circ}$—*Continued*

B.D.		Place 1900				Photographic			Visual			Color-Index	Spectrum	Differences	
		R.A.			Dec.	Mag.	p.e.	No. pl.	Mag.	p.e.	No. pl.			S−Pa	MK−Pa
		h	m	s								M		M	M
74°	456	11	8	41	1′.0	8.34	.09	2	7.24	.04	2	......	G–K±	......	+ .33
74	456a		8	43	1.0				7.68	.00	2	......		......	+ .16
79	*356*		8	48	51.3	8.14	.13	2	6.87	.02	3	+1.27	G5	......	− .11
75	*438*		10	36	53.1	8.55	.06	3	7.46	.01	2	+1.09	K0	......	+ .02
78	*385*		15	58	55.3	7.84	.01	3	7.31	.04	3	+ .53	F5	......	− .01
85	183		24	23	15.5	7.98	.02	3	7.07	.02	4	+ .91	G1	− .13	− .03
s81	373	11	24	48	40.7	6.24	.03	8	6.10	.02	6	+ .14	A2	+ .02	− .02
86	170		28	20	10.1	7.48	.01	10	7.12	.02	3	+ .36	A8	− .01	+ .06
78	392		31	26	8.9	8.07	.16	2	6.34	.03	3	+1.73	K5	......	− .06
73	532		39	0	42.1	8.47	.03	2	6.82	.04	2	+1.65	K5	......	.00
76	*434*		45	23	54.8	8.23	.02	5	7.84	.01	2	+ .39	F±	......	+ .08
74	*475*	11	47	56	50.9	8.42	.06	4	7.44	.06	4	+ .98	G3	......	− .03
74	476		48	19	19.0	7.23	.03	3	6.65	.00	2	+ .58	F6	......	− .01
80	370		54	32	9.1	8.22	.11	3	7.70	.06	5	+ .52	F–G±	− .20	+ .04
81	389		55	6	24.7	7.88	.06	3	6.24	.05	4	+1.64	K8	− .16	− .07
86	176		59	43	8.5	6.68	.01	9	6.15	.03	4	+ .53	F5	+ .01	+ .11
77	460	12	0	2	19.3	7.64	.04	5	7.18	.05	4	+ .46	F2	......	+ .04
77	461		0	9	27.9	6.83	.03	6	5.68	.02	3	+1.15	G3	......	− .04
74	484		0	12	0.6	8.53	.04	4	7.24	.04	3	+1.29	G±	......	+ .04
75	469		4	56	13.1	6.60	.02	5	6.21	.02	3	+ .39	F5	......	+ .08
78	*406*		5	6	56.7	7.77	.05	6	6.74	.04	4	+1.03	G1	......	+ .03
82	356	12	6	30	16.0	7.29	.05	3	6.06	.02	3	+1.23	K2	+ .05	− .16
*82	357		7	3	16.3	....	...	..	8.29	...	1	......	....	......	......
s*78*	*411*		7	6	59.8	6.54	.04	6	6.64	.06	4	− .10	A0	......	+ .07
s78	412		7	31	10.3	5.40	.03	4	5.20	.06	4	+ .20	A4	......	− .14
84	269		8	47	3.6	8.44	.06	4	8.16	.01	4	+ .28	F±	+ .18	.00

73	549	12	11	0	6.5	7.39	.06	3	6.24	.03	2	+1.15	G3	......	− .12
87	*107*		13	54	59.5	6.54	.01	13	6.12	.03	10	+ .42	Fo	+ .05	+ .08
s75	470		14	21	42.9	5.28	.03	4	5.50	.03	2	− .22	B7	......	− .06
88	71		14	23	15.3	6.36	.03	7	6.07	.01	8	+ .29	Fo	+ .03	+ .15
84	*274*		16	32	55.8	8.26	.12	3	7.79	.02	4	+ .47	G±	......	− .13
77	475	12	27	3	14.3	8.60	.03	6	7.22	.06	5	+1.38	G±	......	− .04
75	473		28	2	21.8	8.39	.03	7	7.22	.02	3	+1.17	Go	......	+ .08
73	559		29	23	1.9	7.97	.03	7	7.28	.02	2	+ .69	F±	......	+ .10
76	*454*		29	48	45.8	8.32	.02	6	6.87	.03	3	+1.45	G5±	......	+ .01
81	*400*		31	7	48.1	7.45	.03	5	7.23	.03	6	+ .22	A6	+ .08	− .19
80	*389*	12	34	8	45.9	7.50	.04	4	7.00	.03	4	+ .50	F–Gp	......	− .08
s86	182		34	37	16.9	7.28	.01	10	7.12	.04	4	+ .16	A3	− .04	+ .01
73	561		35	32	33.1	8.24	.02	6	7.08	.02	3	+1.16	Go	......	+ .08
76	462		36	57	27.3	8.48	.03	8	7.08	.02	3	+1.40	K±	......	+ .05
84	286		37	47	11.6	7.72	.02	5	7.24	.01	4	+ .48	F5p	− .05	− .06
s81	402	12	41	54	10.2	6.39	.01	6	6.28	.03	7	+ .11	A3	− .03	+ .02
80	395		42	19	28.2	8.80	.11	3	7.43	.02	6	+1.37	Ko	......	− .01
84	*289*		48	16	57.7	4.62	.05	5	5.64	.13	3	......	A±	+ .16	+ .19
84	*290*		48	23	57.4				5.14	.06	2				+ .22
76	*470*		51	7	57.1	7.78	.01	9	6.84	.00	2	+ .94	Go	......	− .04
79	407	12	52	45	2.5	9.20	.04	2	7.51	.06	3	+1.69	K–M	......	+ .01
83	369		53	9	3.6	8.11	.07	4	6.83	.04	5	+1.28	G–K±	− .10	+ .11
76	473		55	50	.07	7.05	.02	10	6.08	.00	2	+ .97	G2	......	− .22
81	412		58	38	24.8	7.91	.03	4	7.15	.02	6	+ .76	Go	+ .10	+ .03
86	187		59	43	25.4	8.07	.05	7	7.70	.04	4	+ .37	F2	− .24	− .01
s73	583	13	1	44	33.6	6.57	.01	9	6.52	.04	2	+ .05	A3	......	− .28
s73	587		10	40	19.8	6.65	.01	9	6.54	.01	2	+ .11	Ao	......	− .06
80	404		11	12	10.9	8.70	.07	3	7.36	.01	5	+1.34	Ko	......	− .10
81	416		11	32	0.0	7.21	.01	8	6.16	.03	7	+1.05	G3	− .06	− .05
79	418		13	36	13.7	8.46	.04	3	7.45	.03	4	+1.01	Go	......	− .03

CATALOGUE OF ZONE +73° TO +90°—*Continued*

B.D.		Place 1900				Photographic			Visual			Color-Index	Spectrum	Differences	
		R.A.			Dec.	Mag.	p.e.	No. pl.	Mag.	p.e.	No. pl.			S−Pa	MK−Pa
		h	m	s								M		M	M
85°	222	13	18	40	16.′6	7.68	.02	7	7.28	.03	4	+ .40	F5p	− .07	− .20
79	422		26	6	9.6	6.42	.02	4	5.75	.04	3	+ .67	G1	......	− .12
84	*311*		26	42	49.4	7.82	.03	5	7.12	.03	4	+ .70	F9	− .05	+ .02
75	507		27	43	23.8	8.57	.02	4	7.39	.07	4	+1.18	G±	......	− .01
76	491		31	14	34.5	8.09	.02	6	7.01	.04	4	+1.08	G0	......	+ .09
86	*193*	13	32	26	47.1	8.03	.03	7	7.96	.03	4	+ .07	F5p	+ .07	− .12
s76	492		32	36	18.5	7.72	.02	8	7.75	.04	4	− .03	A0	......	+ .01
77	*515*		32	36	48.4	8.75	.06	5	7.81	.03	5	+ .94	G±	......	+ .09
77	516		33	24	3.4	8.10	.02	7	6.31	.02	5	+1.79	K5	......	− .11
80	*417*		36	39	51.7	8.69	.15	2	7.30	.02	4	+1.39	K±	......	+ .02
s77	519	13	39	42	20.9	6.67	.01	11	6.58	.03	5	+ .09	A3	......	+ .07
78	466		42	13	33.9	6.86	.01	4	5.79	.03	3	+1.07	G8	......	− .03
80	421		42	16	42.4	8.53	.05	3	6.86	.05	4	+1.67	K5	− .45	− .07
83	397		45	10	15.3	6.85	.02	6	5.90	.03	4	+ .95	G3	+ .04	− .11
76	500		46	21	4.9	8.49	.01	5	7.99	.06	3	+ .50	A–F	......	+ .13
s76	.502	13	48	27	3.7	7.67	.02	6	7.44	.01	3	+ .23	A5	......	.00
80	422		49	55	24.9	8.17	.04	4	7.12	.05	4	+1.05	G5	− .10	− .06
79	431		50	22	29.3	7.72	.03	4	6.48	.03	3	+1.24	G8	......	− .05
85	234		51	29	0.6	8.71	.12	5	7.65	.02	4	+1.06	G4	......	− .08
81	452		52	27	15.6	7.80	.03	4	6.50	.08	4	+1.30	K2	− .12	.00
74	*560*	13	52	50	48.2	8.53	.03	2	7.60	.04	3	+ .93	G±	......	.00
77	*523*		54	38	59.4	9.1 ±	...	2	7.76	.08	3	+1.3±	K±	......	− .02
74	*563*		58	30	52.9	8.24	.01	3	7.61	.02	3	+ .63	F–G	......	+ .18
86	201		59	25	14.1	7.59	.03	6	7.12	.03	3	+ .47	F2	+ .08	+ .10

75	527	14 4 40	11.8	8.47	.03	3	7.25	.04	4	+1.22	G5	......	+ .05
s75	529	6 9	4.0	6.43	.02	6	6.29	.04	4	+ .14	A2	......	+ .03
78	478	9 14	1.0	6.37	.04	4	4.78	.01	2	+1.59	K5	......	− .12
75	532	21 11	30.9	8.60	.04	2	7.57	.04	3	+1.03	G0	......	− .09
76	527	27 44	8.4	5.66	.03	7	4.31	.02	3	+1.35	K5	......	− .22
79	447	14 28 18	56.5	7.50	.03	4	7.11	.01	2	+ .39	F3	......	+ .11
81	482	32 57	15.2	7.65	.02	4	6.73	.04	5	+ .92	G0	− .14	+ .01
80	448	36 23	5.5	7.59	.01	4	6.11	.02	4	+1.48	K3	.00	+ .04
80	451	41 57	12.8	8.73	.09	3	7.05	.03	4	+1.68	K5±	−1.37	.00
s76	536	48 9	27.4	7.48	.02	6	7.32	.05	3	+ .16	A3	......	+ .02
86	217	14 49 39	21.8	8.03	.03	6	7.01	.05	3	+1.02	G3	− .04	− .03
74	595	51 0	33.8	....	...	..	2.0†	...	..	......	......	......	......
s81	495	54 58	9.3	7.16	.01	8	7.13	.03	5	+ .03	A1	− .07	− .01
78	501	55 25	34.8	7.52	.03	3	6.45	.03	3	+1.07	G5	......	− .08
75	545	55 41	17.0	8.60	.08	3	6.86	.01	2	+1.74	K–M	......	− .08
83	431	14 57 3	55.4	6.20	.01	6	5.49	.02	4	+ .71	F8	+ .04	+ .10
75	547	57 31	18.0	7.89	.05	4	6.76	.01	2	+1.13	G0	......	− .12
84	335	15 1 41	20.3	8.31	.04	5	6.72	.03	4	+1.59	K2	− .21	− .02
s74	602	5 39	16.5	7.68	.02	3	7.49	.01	2	+ .19	A3	......	− .03
87	143	9 21	37.1	8.37	.04	4	6.74	.03	8	+1.63	K5	− .36	+ .06
74	609	18 15	24.4	7.87	.03	5	6.60	.05	3	+1.27	G5	......	− .10
78	510	15 21 30	45.2	8.62	.01	2	7.12	.15	2	+1.50	K±	......	.00
73	672	22 18	49.7	7.84	.03	5	7.34	.02	3	+ .50	F+	......	− .04
73	680	31 33	16.5	8.72	...	1	7.72	.01	2	+1.00	......	......	+ .17
77	592	34 23	40.9	6.63	.04	9	4.84	.03	4	+1.79	K5	......	− .02
80	480	34 59	46.8 }	6.88	.02	6	6.56	.08	3	......	G2	− .12	+ .03
80	481	35 12	46.8 }				7.30	.03	3	......			......

† β *Ursae Minoris*, too bright

CATALOGUE OF ZONE $+73°$ TO $+90°$—*Continued*

B.D.		Place 1900 R.A.			Place 1900 Dec.	Photographic Mag.	Photographic p.e.	Photographic No. pl.	Visual Mag.	Visual p.e.	Visual No. pl.	Color-Index	Spectrum	Differences S−Pa	Differences MK−Pa
		h	m	s								M		M	M
81°	517	15	35	57	6′.2	7.92	.03	3	6.86	.04	3	+1.06	G2	− .18	− .02
s76	563		36	50	46.7	7.62	.02	7	7.51	.02	2	+ .11	A±	......	+ .17
82	463		38	8	35.8	7.25	.03	7	7.01	.03	2	+ .24	F0	− .04	− .17
85	263		42	32	9.4	7.88	.03	7	6.79	.03	4	+1.09	G1	− .09	− .09
81	523	15	42	56	56.0	7.83	.08	4	6.85	.04	3	+ .98	G5±	− .04	− .11
80	487		45	7	17.8	7.20	.02	6	6.78	.02	3	+ .42	F0	− .14	+ .01
78	527		47	37	6.1	4.11	.14	2	4.09	.08	2	+ .02	A1	......	+ .15
81	531		51	24	14.2	8.30	.04	3	6.79	.05	3	+1.51	K5	......	− .11
s83	453		53	46	15.0	7.61	.03	7	7.41	.06	3	+ .20	A3	− .04	+ .04
75	579	15	57	23	51.7	7.82	.02	7	6.73	.08	3	+1.09	G5	......	− .03
s85	269		57	25	35.3	7.07	.02	10	6.82	.02	5	+ .25	A5	− .12	+ .06
76	580	16	0	19	22.1	8.67	.06	5	7.90	.04	3	+ .77	G±	......	+ .12
74	650		1	56	12.6	8.95	.05	2	7.70	.03	2	+1.25	K±	......	+ .09
73	707		5	24	24.8	7.35	.04	6	6.81	.05	2	+ .54	F5	......	− .08
s77	616		6	50	3.6	5.59	.02	6	5.73	.01	3	− .14	A1	......	− .05
81	*541*		7	29	53.8	8.41	.01	4	7.44	.03	3	+ .97	G±	− .66?	+ .01
s76	594	16	13	40	7.8	5.18	.02	5	5.52	.02	2	− .34	A0	......	− .14
75	586		15	3	27.5	7.79	.03	4	6.16	.01	2	+1.63	K5	......	.00
s73	713		16	12	38.3	5.94	.02	8	6.08	.02	2	− .14	A±	......	− .06
76	*596*		20	26	59.0	5.18	.04	8	5.10	.05	2	+ .08	A6	......	− .30
77	623		22	39	47.1	8.97	.04	3	7.42	.06	3	+1.55	K±	......	− .04

s76	600	16 22 59	22.5	7.23	.02	10	6.92	.01	2	+ .31	A3	......	+ .07
76	606	25 58	40.2	9.0±	...	2	7.28	.03	2	+1.7±	..	......	− .08
79	498	31 19	10.6	5.67	.01	4	5.61	.01	2	+ .06	A3	......	− .04
s*84*	*361*	33 37	55.1	7.29	.02	6	7.03	.03	6	+ .26	A4	− .06	+ .08
77	627	34 57	38.6	7.43	.03	8	6.41	.05	3	+1.02	G4	......	− .27
80	*519*	16 37 45	59.7	7.53	.01	5	6.90	.04	3	+ .63	F–G	......	− .03
78	*562*	39 55	57.1	7.49	.02	7	6.98	.03	3	+ .51	F5	......	− .36
79	511	43 34	6.4	7.50	.03	3	6.33	.07	2	+1.17	K2p	......	− .17
s74	680	44 10	4.2	6.99	.01	11	6.74	.07	2	+ .25	A2	......	+ .08
77	634	47 32	41.1	6.36	.02	8	6.13	.06	3	+ .23	F1	......	− .13
75	605	16 52 37	32.6	7.87	.03	6	7.34	.05	2	+ .53	F±	......	+ .08
79	517	54 6	40.1	8.08	.01	2	6.77	.05	4	+1.31	K1	......	− .16
s75	608	56 1	33.0	7.03	.01	7	6.78	.05	2	+ .25	A5	......	+ .04
82	498	56 12	12.1	5.05	.03	8	4.46	.04	4	+ .59	G2	+ .15	− .26
s77	639	56 49	0.5	7.61	.03	9	7.36	.01	2	+ .25	A5	......	− .05
s73	751	16 58 16	16.8	6.47	.01	6	6.30	.01	2	+ .17	A5	......	− .08
74	695	58 49	26.1	7.53	.03	4	6.98	.08	2	+ .55	F3	......	+ .14
78	573	17 0 52	6.4	8.42	.07	2	7.59	.02	2	+ .83	G±	......	− .31
s77	641	0 55	48.0	6.60	.02	10	6.86	.00	2	− .26	B9	......	− .11
73	754	2 31	20.1	8.60	.02	3	7.23	00	2	+1.37	..	......	+ .06
73	755	3 24	27.1	9.0±	...	1	7.54	.01	2	+1.4±	..	......	− .03
75	612	3 31	22.0	7.29	.02	7	6.84	.01	2	+ .45	F0	......	+ .06
81	568	17 4 46	0.1	7.70	.02	6	6.59	.03	5	+1.11	G5	− .09	− .07
s75	613	4 49	26.2	6.37	.01	7	6.24	.02	2	+ .13	A5	......	− .11
78	580	5 15	14.2	8.40	.03	2	7.06	.06	3	+1.34	G–K±	......	− .20
75	616	9 7	13.8	7.89	.02	5	7.36	.08	3	+ .53	F±	......	− .04
s75	617	13 55	12.7	8.14	.03	5	8.06	.07	3	+ .08	A±	......	+ .12
77	654	17 19 5	27.3	8.70	.04	5	7.86	.03	3	+ .84	G±	......	− .08
s74	710	25 36	44.9	7.76	.02	5	7.49	.03	3	+ .27	A5	......	+ .04
s79	540	25 49	23.8	7.35	.02	3	7.30	.03	2	+ .05	A1	......	− .01
80	544	27 12	13.5	7.32	.04	5	5.56	.05	4	+1.76	K5+	+ .03	− .08
76	647	27 22	8.4	8.56	.03	6	7.19	.03	3	+1.37	K±	......	− .17

CATALOGUE OF ZONE $+73^\circ$ TO $+90^\circ$—*Continued*

B.D.		Place 1900 R.A.	Place 1900 Dec.	Photographic Mag.	Photographic p.e.	Photographic No. pl.	Visual Mag.	Visual p.e.	Visual No. pl.	Color-Index	Spectrum	Differences S−Pa	Differences MK−Pa
		h m s								M		M	M
83°	512	17 32 37	24′.9	8.11	.02	3	7.52	.07	4	+ .59	F+	− .19	− .18
74	717	35 27	17.4	7.94	.02	3	6.77	.04	2	+1.17	K±	......	− .19
74	720	38 47	3.8	8.30	.02	2	7.72	.08	2	+ .58	F-G±	......	− .10
75	639	42 31	57.7	9.1±	...	1	8.06	.03	2	+1.00±	..	......	− .01
79	556	44 9	15.5	7.67	.02	2	7.40	.04	2	+ .27	A8	......	− .20
s73	791	17 44 26	30.1	7.83	.01	2	7.76	.00	2	+ .07	A2	......	− .04
75	640	46 8	34.6	8.76	...	2	7.82	.02	2	+ .96	..	......	− .11
73	796	49 50	9.8	8.35	.06	2	7.75	.08	2	+ .60	F±	......	− .13
80	555	50 5	18.9	8.59	.03	2	6.91	.04	5	+1.68	K5+	......	− .03
77	670	52 59	3.0	7.98	.02	5	7.57	.04	3	+ .41	F±	......	+ .10
76	667	53 55	58.5	5.32	.02	8	5.01	.01	2	+ .31	F5?	......	+ .13
74	740	17 54 22	35.2	6.92	...	1	6.77	.04	3	+ .15	A5	......	− .05
79	564	55 35	21.0	7.77	.00	2	7.52	.02	2	+ .25	A8	......	− .03
78	616	55 49	19.4	7.72	.03	2	6.16	.04	3	+1.56	K5	......	− .12
s76	668	56 8	0.8	7.62	.03	5	7.33	.02	3	+ .29	A6	......	− .03
75	647	57 45	10.8	7.42	.02	4	6.56	.03	3	+ .86	G5	......	− .28
78	621	18 0 6	41.2	7.81	.03	2	7.62	.06	3	+ .19	F±	......	.00
s86	269	4 33	36.8	4.25	.04	6	4.41	.02	17	− .16	B9	+ .09	− .01
s75	655	6 31	46.6	6.91	.01	5	6.98	.03	3	− .07	A1	......	− .03
85	294	7 11	40.9	7.84	.03	6	7.59	.02	4	+ .25	A7	− .13	+ .04
79	570	7 31	59.2 }	5.39	.02	8	5.58	.16	3	......	F±	......	+ .44
79	571	7 37	59.4 }				5.79	.04	2	......		......	− .11
s86	272	18 7 48	59.6	5.91	.03	4	5.73	.01	18	+ .18	A0	+ .01	+ .08
76	678	9 58	6.7	8.12	.03	4	7.66	.05	2	+ .46	F±	......	− .19
77	681	12 24	34.1	8.32	.10	2	7.41	.04	4	+ .91	G±	......	− .15
74	757	14 53	18.1	>8.7−	...	2	7.84	.03	2	......	..	......	− .08

81	622	18	22	26	26.0	8.75	.02	3	7.18	.02	4	+1.57	K±	......	− .14
s82	540		22	56	53.9	7.25	.01	6	7.45	.05	3	− .20	B8	− .15	− .15
84	412		24	34	37.1	7.71	.01	6	7.27	.03	4	+ .44	F2	− .07	− .09
*86	275		26	18	33.4	8.81	.03	3	7.65	.01	4	+1.16	G5	......	......
77	696		29	2	29.7	8.03	.01	3	6.98	.04	5	+1.05	G5	......	− .06
79	587	18	30	37	9.2	7.79	.11	3	6.45	.05	4	+1.34	K0	......	− .02
79	590*a*		33	4	33.3	7.58	.09	4	7.20	.04	3	+ .38	A7	......	− .08
77	699		34	35	28.1	6.92	.04	6	5.59	.09	5	+1.33	K2	......	− .07
83	535		36	27	17.8	8.33	.09	4	7.20	.07	3	+1.13	G3	− .34	− .04
s83	536		37	22	6.1	6.14	.02	8	6.17	.02	4	− .03	A0	.00	− .02
73	831	18	37	57	11.5	8.28	.02	2	7.36	.02	3	+ .92	G±	......	− .04
77	702		42	17	35.4	7.25	.06	5	6.87	.04	5	+ .38	A7	......	− .03
s75	678		44	37	11.6	7.73	.04	4	7.46	.02	3	+ .27	A6±	......	+ .04
86	282		47	45	34.8	8.29	.01	5	6.60	.04	3	+1.69	M	− .37	− .24
73	835		48	16	58.2	6.21	.02	6	5.44	.05	2	+ .77	G±	......	− .26
75	682	18	49	36	19.0	5.28	.01	7	5.49	.02	3	− .21	B8	......	− .12
74	792		49	45	36.4	7.94	.02	4	7.18	.01	2	+ .76	G±	......	− .10
s79	604		52	41	49.3	6.71	.05	8	6.39	.06	4	+ .32	A4	......	− .07
78	660		55	43	42.5	8.53	.06	3	7.99	.08	4	+ .54	F±	......	− .03
78	661		56	8	50.2	8.22	.07	3	7.84	.05	3	+ .38	A6	......	− .11
75‡	683	18	56	54	39.2	6.24	.02	5	6.39	.01	2	− .15	A0	......	......
s73	845	19	1	2	59.5	7.31	.02	4	7.09	.01	2	+ .22	A5	......	− .08
76	712		2	10	54.5	7.03	.04	7	6.65	.00	2	+ .38	A9	......	− .21
s83	547		4	1	46.2	6.92	.02	9	6.83	.04	5	+ .09	A2	− .11	+ .03
s82	572		4	41	13.6	6.85	.03	5	6.88	.02	4	− .03	B9	+ .02	+ .06
80	604		6	4	17.9	8.97	.06	2	7.90	.06	4	+1.07	G±	......	− .02
77	715	19	6	26	31.4	9.13	.08	2	7.54	.03	4	+1.59	K±	......	+ .04
79	614		11	23	29.1	8.80	.06	3	7.96	.01	2	+ .84	F–G±	......	− .06
76	717		12	50	23.7	5.29	.04	8	5.43	.04	2	− .14	A5	......	− .43
80	607		14	22	33.6	8.35	.05	4	7.28	.06	4	+1.07	G0	− .66	− .03
s80	609		15	32	34.9	7.77	.01	4	7.57	.04	3	+ .20	A2	+ .14	+ .07

‡ Σ 2452, dist. = 5″.6.

CATALOGUE OF ZONE $+73°$ TO $+90°$—*Continued*

B.D.		Place 1900				Photographic			Visual			Color-Index	Spectrum	Differences	
		R.A.			Dec.	Mag.	p.e.	No. pl.	Mag.	p.e.	No. pl.			S−Pa	MK−Pa
		h	m	s								M		M	M
73°	857	19	17	29	10′.2	5.84	.02	7	4.92	.11	2	+ .92	Ko	......	− .78
s73	860		20	51	21.5	7.09	.05	7	7.04	.02	2	+ .05	A5	......	− .11
88	112		22	30	59.3	8.09	.03	7	6.23	.02	12	+1.86	M	− .30	− .04
76	732		22	36	36.0	8.55	.02	5	7.30	.01	2	+1.25	G5	......	− .21
76	734		25	7	21.7	9.06	.03	3	6.34	.03	2	+2.72	N	......	− .38
s79	628	19	27	45	24.2	6.05	.01	4	6.17	.03	3	− .12	A2	......	− .04
s83	552		27	57	16.1	6.60	.02	7	6.43	.03	4	+ .17	A2	− .07	+ .07
73	863		28	44	9.4	7.87	.06	5	7.68	.01	2	+ .19	A5±	......	− .12
81	666		28	46	35.8	9.02	.17	2	7.62	.03	4	+1.40	K±	......	+ .12
77	734		34	8	2.9	7.44	.04	6	{ 7.36	.05	4	......	A6±	......	+ .36
									{ 7.99	...	1	......		......	+ .36
74	831	19	37	16	9.0	8.11	.06	4	6.96	.04	2	+1.15	G5	......	− .17
79	638		39	59	56.0	8.75	.02	2	8.46	.06	4	+ .29	F±	......	− .18
79	645		49	15	17.4	7.77	.05	3	7.96	.04	4	− .19	Ao	......	− .18
79	648		51	42	12.4	8.38	.02	4	7.32	.06	4	+1.06	G+	......	− .12
78	694		55	35	21.6	9.1±	...	3	7.62	.02	5	+1.5±	..	......	− .16
75	718		59	23	26.4	8.72	.15	2	7.22	.02	3	+1.50	K5±	......	− .14
76	771	20	2	25	12.2	7.95	.04	7	6.17	.07	3	+1.78	K5	......	− .12
73	897		5	51	37.2	7.77	.02	4	6.82	.00	2	+ .95	G2	......	+ .04
79	660		7	37	23.5	7.63	.04	5	6.76	.02	5	+ .87	Go	......	− .21
s77	764		12	16	24.6	4.22	.03	4	4.63	.04	7	− .41	B5	......	− .28
85	340		13	38	28.3	9.7±	...	2	7.72	.06	3	+2.0±	K+	......	− .20
s84	451	20	13	59	22.6	6.96	.03	6	6.79	.06	4	+ .17	A2	− .09	.00
81	699		15	33	54.8	7.83	.05	3	7.35	.05	2	+ .48	Fp	+ .05	+ .05
79	668		18	11	20.5	8.24	.03	3	6.81	.04	4	+1.43	K±	......	− .22
s80	650		20	9	13.1	6.69	.01	9	6.80	.03	5	− .11	B9	+ .04	+ .03
84	462		24	28	13.7	7.47	.01	5	7.19	.06	4	+ .28	A8	+ .01	− .15

84	463	20	24	33	48.7	7.62	.02	4	7.03	.05	4	+ .59	F5	− .05	− .04
75	739		24	50	43.0	9.15	.05	2	7.68	.03	3	+1.47	G–K±	......	− .01
81	*706*		28	43	2.5	7.43	.02	5	6.89	.05	2	+ .54	F2	− .01	+ .03
79	675		30	34	53.1	7.40	.03	7	7.54	.05	4	− .14	A0	......	− .06
s74	872		32	50	36.7	5.24	.06	5	5.12	.11	4	+ .12	A±	......	− .06
80	*657*	20	33	9	5.7	7.59	.04	6	6.49	.01	4	+1.10	G4	+ .07	− .04
82	617		34	23	49.7	7.72	.01	4	6.67	.06	3	+1.05	G2	− .08	− .10
80	*659*		34	32	4.8	6.53	.01	10	5.48	.04	4	+1.05	G4	− .07	− .38
80	660		35	15	44.5	6.97	.02	7	5.97	.05	4	+1.00	G2	− .03	+ .01
75	752		38	39	13.7	8.35	.08	3	7.34	.03	4	+1.01	G+	......	+ .08
s83	588	20	39	5	16.8	6.37	.01	6	6.21	.02	5	+ .16	A2	− .10	+ .06
78	*716*		39	57	4.6	6.65	.02	8	6.97	.04	4	− .32	B2	......	+ .04
s76	809		40	10	28.5	7.38	.01	9	7.03	.02	4	+ .35	F±	......	+ .06
81	712		41	35	39.3	8.11	.02	5	6.92	.03	4	+1.19	K±	......	− .08
s*81*	*718*		49	51	9.7	5.78	.02	10	5.81	.05	4	− .03	B8	− .07	+ .07
80	672	20	52	8	10.6	6.67	.02	10	5.49	.03	3	+1.18	G8	− .09	− .37
74	890		52	34	15.6	7.29	.04	4	7.17	.02	3	+ .12	A2±	......	+ .01
75	764		55	55	32.3	6.98	.02	3	5.91	.02	4	+1.07	G2	......	+ .12
75	765		57	13	19.8	8.03	.02	2	6.62	.02	4	+1.41	K5	......	+ .16
73	922	21	0	4	53.5	>8.9±	...	2	8.01	.03	3	>+ .9 ±	G±	......	− .17
s77	800		7	30	43.2	5.83	.04	10	6.03	.04	6	− .20	A0	......	− .13
80	679		8	6	45.4	7.17	.02	8	6.92	.03	6	+ .25	A6	− .03	− .05
74	907		10	6	49.1	7.44	.01	2	6.80	.02	3	+ .64	F5	......	− .08
80	682		11	4	36.7	8.35	.06	3	6.99	.05	5	+1.36	G–K±	......	− .15
78	742	21	11	22	15.1	7.44	.02	5	7.46	.02	3	− .02	A0	......	− .02
s75	778		12	57	53.7	6.85	.01	6	7.00	.02	3	− .15	B8	......	− .04
78	744		13	49	33.6	7.04	.01	5	7.17	.04	3	− .13	B5	......	− .19
80	688		16	45	23.2	7.37	.05	6	7.36	.02	3	+ .01	A1	......	− .12
76	833		16	48	35.5	7.63	.02	7	5.92	.03	3	+1.71	K5–M	......	− .15

CATALOGUE OF ZONE $+73^\circ$ TO $+90^\circ$—*Continued*

B.D.		Place 1900				Photographic			Visual			Color-Index	Spectrum	Differences	
		R.A.			Dec.	Mag.	p.e.	No. pl.	Mag.	p.e.	No. pl.			S−Pa	MK−Pa
		h	m	s								M		M	M
s80°	690	21	17	31	48′.7	6.21	.01	10	6.05	.03	6	+ .16	A2	− .06	+ .07
77	*811*		17	33	10.6	7.13	.04	7	7.43	.03	3	− .30	A0	......	+ .03
s86	319		19	35	37.4	7.41	.02	5	7.43	.05	4	− .02	A2	− .12	− .05
83	603		21	31	50.3	8.10	.03	5	6.80	.04	3	+1.30	K±	− .41	− .06
81	737		22	59	5.2	8.24	.04	5	7.80	.07	3	+ .44	F±	......	− .01
s76	836	21	23	18	39.6	6.62	.02	14	6.55	.03	4	+ .07	A0	......	− .05
75	*787*		23	27	7.2	7.43	.00	3	6.79	.03	4	+ .64	F5	......	− .05
75	788		24	46	32.3	8.35	.06	3	7.19	.02	4	+1.16	G5	......	− .15
79	*707*		27	46	5.4	6.79	.03	7	6.01	.04	3	+ .78	G1	......	− .03
75	701		28	54	57.9	7.97	.05	4	7.66	.11	4	+ .31	A7	......	− .21
77	823	21	29	55	29.7	8.33	.05	6	7.19	.03	5	+1.14	G5	......	− .08
s*72*	*991*		30	3	2.1	8.54	.02	2	8.03	.06	3	+ .51	A–F±	......	− .19
s74	926		37	9	46.2	8.13	.03	2	7.92	.01	3	+ .21	A2	......	+ .04
s77	834		46	33	46.1	7.02	.01	9	7.02	.02	3	+ .00	A0	......	+ .08
s83	618		50	21	34.2	7.35	.06	3	7.08	.02	5	+ .27	A4	− .23	− .04
77	836	21	50	57	17.9	8.7±	...	2	7.77	.07	4	+ .9±	G±	......	− .09
s*72*	*1003*		51	37	13.7	6.68	.01	4	6.73	.05	2	− .05	A0	......	− .17
78	*768*		53	19	4.7	7.95	.03	4	6.46	.02	2	+1.49	K5	......	− .10
79	721		55	54	50.0	7.88	.02	5	6.39	.02	3	+1.49	M	......	− .11
74	946		56	54	31.1	8.05	.03	2	6.47	.07	2	+1.58	K5	......	− .33
75	808	21	57	20	36.6	8.66	...	1	7.75	.09	3	+ .91	G±	......	+ .05

82	673	22	1	52	23.3	7.04	.02	6	7.02	.02	3	......	F5	......	− .11
82	674		1	58	23.4				7.39	.09	3			......	− .15
75	818		11	28	57.9	8.34	.03	3	6.88	.08	3	+1.46	G–K±	......	− .18
s75	820		17	8	59.1	6.50	.02	6	6.62	.08	2	− .12	B7	......	− .02
s75	822		17	43	30.6	7.62	.07	2	7.42	.01	2	+ .02	A1	......	+ .02
s83	*630*	22	20	54	0.1	7.50	.02	2	7.46	.02	4	+ .04	A1	− .09	+ .02
s85	383		21	18	36.3	5.06	.04	9	5.24	.02	20	− .18	B8	+ .08	− .15
85	384		21	41	43.2	7.87	.01	4	6.49	.02	8	+1.38	K0	− .26	− .05
s77	860		22	52	44.1	6.65	.01	8	6.68	.07	4	− .03	B8	......	+ .10
81	775		23	42	25.7	7.43	.03	4	6.90	.04	3	+ .53	F3	......	+ .12
s76	859	22	23	59	55.4	7.49	.02	5	7.36	.07	2	+ .13	A5	......	− .14
s87	205		24	16	34.4	7.48	.02	7	7.43	.06	6	+ .05	A2	− .11	− .17
78	796		25	57	16.6	6.02	.01	5	5.63	.07	2	+ .39	A2	......	+ .21
79	*739*		26	8	11.4	7.75	.01	4	6.56	.07	3	+1.19	G6	......	− .06
84	509		27	29	33.2	8.08	.01	4	7.16	.07	3	+ .92	G2	− .21	− .14
78	801	22	29	0	18.7	5.58	.03	5	5.31	.00	2	+ .27	A2	......	− .01
s75	836		30	31	42.7	5.58	.03	6	5.71	.03	2	− .13	B8	......	− .11
73	982		30	37	33.4	8.36	.02	2	6.80	.04	2	+1.56	K5	......	− .04
72	*1049*		33	17	7.4	5.43	.04	4	5.15	.02	2	+ .28	F5	......	− .13
74	978		35	4	51.1	7.55	.04	3	5.72	.06	4	+1.83	K5+	......	− .10
80	731	22	39	12	52.2	7.50	.02	4	6.72	.05	3	+ .78	F5	− .05	+ .02
76	*870*		39	48	5.5	7.46	.01	4	7.22	.02	4	+ .24	A4	......	− .02
73	989		40	4	48.5	8.27	.06	2	7.54	.02	3	+ .73	F5±	......	+ .16
73	991		41	51	54.1	>8.7±	...	3	7.54	.02	3	>1.2±	G5±	......	+ .04
s77	871		42	15	59.5	7.17	.03	4	7.31	.08	3	− .14	B8	......	− .03
73	*994*	22	43	4	1.7	8.53	...	3	7.20	.06	3	+1.33	K±	......	− .02
79	750		43	10	54.6	7.88	.03	4	7.06	.00	2	+ .82	G0	......	− .16
82	703		47	53	37.4	6.12	.02	7	4.62	.06	4	+1.50	K2	− .02	− .09
79	756		48	25	50.2	7.93	.05	5	6.98	.04	2	+ .95	G1	......	− .11
84	513		50	3	14.7	8.11	.03	4	7.01	.05	4	+1.10	G5	− .26	− .07

CATALOGUE OF ZONE +73° TO +90°—*Continued*

B.D.		Place 1900 R.A.	Place 1900 Dec.	Photographic Mag.	Photographic p.e.	Photographic No. pl.	Visual Mag.	Visual p.e.	Visual No. pl.	Color-Index	Spectrum	Differences S−Pa	Differences MK−Pa
		h m s								M		M	M
73°	*998*	22 51 18	4′.8	7.86	.01	2	7.28	.02	3	+ .58	F–G	......	+ .17
84	517	53 29	50.3	7.25	.02	5	5.74	.04	5	+1.51	K4	+ .02	+ .01
77	879	54 37	57.7	8.25	.10	2	7.53	.04	3	+ .72	G±	......	+ .35
83	640	55 12	48.7	6.19	.02	6	4.57	.07	4	+1.62	K6	+ .03	− .09
79	759	57 24	48.4	8.25	.05	3	7.22	.08	3	+1.03	G5	......	− .34
75	867	22 58 11	35.1	8.28	...	1	7.32	.00	2	+ .96	F–G±	......	− .02
s 76	892	58 25	19.9	7.94	.03	3	7.70	.03	2	+ .24	A+	......	+ .10
79	*761*	59 32	14.6	6.91	.02	7	6.69	.02	3	+ .22	A5	− .13	− .07
s *74*	*1002*	23 3 11	2.4	6.64	.03	3	6.62	.04	3	+ .02	A0	......	+ .05
74	1006	4 43	50.8	5.11	.02	4	4.36	.04	3	+ .75	F8	......	− .10
79	*769*	5 29	1.7	7.34	.05	4	7.22	.06	3	+ .12	A3	− .13	− .12
s 73	1023	11 4	41.1	5.75	.01	6	5.78	.02	3	− .03	A0	......	+ .10
78	825	12 46	41.2	7.73	.09	2	8.04	.10	2	− .31	B8±	......	− .24
77	896	23 13 9	36.3	7.61	.02	4	7.35	.03	4	+ .26	A4	......	− .01
s 74	1016	13 47	45.2	6.32	.03	6	6.19	.03	3	+ .13	A0	......	+ .07
72	*1096*	14 17	8.5	8.35	.04	2	7.05	.01	2	+1.30	G–K±	......	− .06
73	1035	23 43	34.1	7.49	.03	4	7.10	.08	3	+ .39	F1	......	− .02
78	*835*	23 57	14.6	7.75	.03	2	7.57	.08	2	+ .18	A2	......	+ .10
85	399	23 24 24	52.1	6.79	.02	5	6.50	.05	5	+ .29	A6	+ .08?	+ .04
s 74	1022	25 2	40.5	6.56	.01	6	6.59	.02	4	− .03	A0	......	− .03
s 77	908	26 50	20.5	7.04	.01	7	7.10	.02	5	− .06	A0	......	− .09
s 86	344	27 48	45.4	5.62	.03	6	5.54	.01	24	+ .08	A4	+ .08	− .02
s 77	909	27 50	16.0	7.06	.02	7	7.03	.01	4	+ .03	A0	......	− .09

82	728	23 33 17	39.0	8.44	.05	2	7.35	.03	3	+1.09	K±	......	− .06
s74	1032	34 54	44.3	5.95	.04	4	5.79	.07	3	+ .16	A3	......	+ .08
73	1047	35 0	26.9	6.94	.05	3	5.91	.02	3	+1.03	G2	......	− .03
76	*928*	35 15	4.4	4.28	.03	4	3.56	.08	5	+ .72	G0	......	− .41
76	*934*	47 8	2.8	6.89	.02	5	6.56	.03	5	+ .33	F1	......	− .10
74	1047	23 47 30	59.2	7.29	.01	2	6.33	.05	3	+ .96	G2	......	− .03
s73	1063	49 58	51.2	6.57	.02	4	6.47	.03	3	+ .10	A0	......	+ .02
74	1051	50 24	19.0	8.63	...	1	7.62	.02	2	+1.01±	G±	......	− .18
73	*1068*	54 10	15.0	8.10	.03	3	7.80	.05	2	+ .30	A8	......	− .05
s*72*	*1135*	56 32	3.3	6.66	.02	3	6.40	.00	2	+ .26	A±	......	+ .07
s82	748	23 57 34	25.0	7.27	.03	2	7.16	.01	3	+ .11	A0	− .05	− .04
79	799	57 36	43.5	7.91	.01	5	7.82	.08	3	+ .09	F0	......	− .31

DISCUSSION OF CATALOGUE RESULTS

Probable error and number of plates.—The photographic images occurred on the average on 4.7 plates. In this reckoning the short duplicate exposures are counted as separate plates. The "visual" images averaged 3.4 plates, there being no duplicate exposures. The probable errors of the catalogue mean of the magnitude results are given in detail in the two following tables.

TABLE II

PROBABLE ERRORS CLASSIFIED BY NUMBER OF PLATES

No. of Plates	Photographic		"Visual"	
	No. of Stars	Probable Error	No. of Stars	Probable Error
2	89	±0.046	177	±0.031
3	116	±0.042	241	±0.041
4	131	±0.033	146	±0.040
5	79	±0.029	49	±0.036
6	90	±0.024	14	±0.035
7	44	±0.024	3	±0.033
8	34	±0.022	3	±0.020
9	11	±0.016	3	±0.037
10	16	±0.018	2	±0.030
11	4	±0.010	...	....
12	...	....	1	±0.020
13	1	±0.010	1	±0.030
14	1	±0.020	...	....

TABLE III

PROBABLE ERRORS CLASSIFIED BY MAGNITUDES

Limiting Magnitudes	Photographic			"Visual"		
	No. of Stars	No. of Plates	Probable Error	No. of Stars	No. of Plates	Probable Error
Brighter than 6.0	33	5.2	±0.035	58	3.5	±0.039
6.00 to 6.49	28	6.4	±0.020	61	4.0	±0.033
6.50 to 6.99	66	6.6	±0.021	104	3.6	±0.037
7.00 to 7.24	30	6.2	±0.024	87	3.3	±0.037
7.25 to 7.49	54	5.5	±0.023	107	3.4	±0.038
7.50 to 7.74	60	4.6	±0.024	73	3.2	±0.032
7.75 to 7.99	74	4.7	±0.030	61	3.1	±0.040
Fainter than 8.0	256	3.7	±0.042	44	3.1	±0.047
Mean of all	...	4.7	±0.032	...	3.4	±0.036

For the photographic magnitudes the stars brighter than 6.0 and fainter than 8.0 have large probable errors. Excepting these

two groups the mean probable error is ±0.024. For the "visual" magnitudes the dependence on brightness is not so evident. Making due allowance for the occurrence of the "visual" images on a less number of plates, the precision of the two methods is about equal.

RELATION BETWEEN SPECTRUM AND COLOR-INDEX

In Table IV the stars are arranged according to spectral type and the mean color-index of each group is taken. Fig. 11 is a

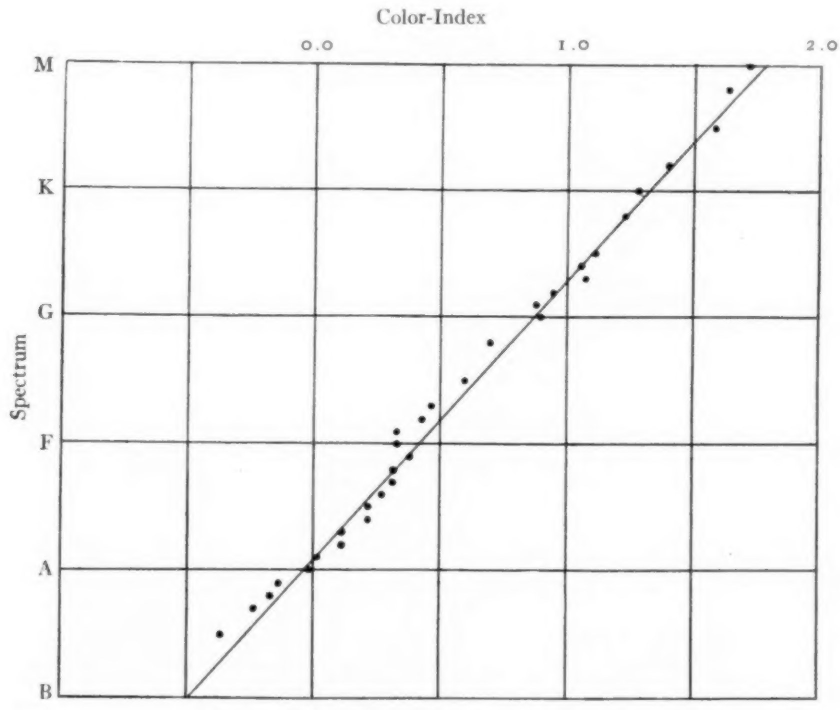


FIG. 11.—Relation between spectrum and color-index

graphic representation of the same data. It will be seen that a straight line fits the platted points as well as any simple curve. True, there is a systematic clustering of the points below the curve between B8 and A8, above between A8 and G, and below between

G and K, but the divergence is slight and the data too meager to warrant the use of a complex curve to represent the relation.

TABLE IV

Limits	No. Stars	Mean Spectrum	Mean Color-Index	Limits	No. Stars	Mean Spectrum	Mean Color-Index
B2 to B6...	5	B5	−0.36	B2 to B6...	11	F2	+0.43
	3	B7	−0.23		4	F3	+0.46
	12	B8	−0.16	F4 to F6	24	F5	+0.60
	10	B9	−0.14	F8 to F9	6	F8	+0.70
					22	G0	+0.90
	55	A0	−0.01		11	G1	+0.88
	22	A1	+0.02		25	G2	+0.95
	30	A2	+0.12		13	G3	+1.08
	23	A3	+0.12		9	G4	+1.06
	17	A4	+0.22		25	G5	+1.12
	20	A5	+0.22	G6 to G9	7	G8	+1.24
	12	A6	+0.27		16	K0	+1.29
	7	A7	+0.31	K1 to K3	13	K2	+1.40
	9	A8	+0.31	K4 to K6	32	K5	+1.59
	1	A9	+0.39		1	K8	+1.64
	20	F0	+0.33		5	M	+1.73
	9	F1	+0.33		1	N	+2.74

There are 492 stars used in Table IV. Their average magnitude is 7.3 visual = 7.7 photographic. Table V shows a comparison

TABLE V

Spectrum	Color-Index		
	Parkhurst	King	Schwarzschild
	M	M	M
B5	−0.28	−0.17	−0.50
A0	−0.06	0.00	−0.30
A5	+0.18	+0.18	−0.10
F0	+0.40	+0.32	+0.10
F5	+0.63	+0.52	+0.30
G0	+0.86	+0.71	+0.54
G5	+1.08	+0.90	+0.80
K0	+1.32	+1.16	+1.05
K5	+1.55	+1.42	+1.50
M	+1.77	+1.67	......

between ten points from the curve in Fig. 11 and the corresponding points from King's curve from 153 bright stars, given in *Harvard Annals*, **59**, 180; also from Schwarzschild's curve from his "Aktino-

metrie" stars in the *Harvard Revised Photometry*, given in Part B, p. 19.

It will be noticed that in the range of color-index included between the extremes of spectra, the present work lies between the other two. The difference of about 10 per cent remaining between King's values and mine seems to be due mainly to the estimates of spectral type. The larger differences with Schwarzschild depend somewhat on the earlier Draper Catalogue estimates of spectra, and are therefore more uncertain.

COMPARISON WITH SCHWARZSCHILD'S PHOTOGRAPHIC MAGNITUDES

There are 151 stars in common with the list of polar stars in the *Aktinometrie*, B, p. 42. These were arranged in four groups according to magnitude, and each group in four or five divisions according to color-index. Table VI gives the differences in the photographic magnitudes in the sense Parkhurst minus Schwarzschild.

TABLE VI

Mean Mag.	Division 1			Division 2			Division 3			Division 4			Division 5		
	Color	Δ Mag.	No.	Color	Δ Mag.	No.	Color	Δ Mag.	No.	Color	Δ Mag.	No.	Color	Δ Mag.	No.
6.50	−.05	+.03	10	+.15	+.01	10	+.33	+.01	11	+1.23	−.03	..	....	.....	..
7.35	−.03	+.05	10	+.16	+.04	9	+.33	+.09	10	+.79	+.05	5	1.30	+.01	5
7.78	+.07	+.05	9	+.41	+.04	10	+1.05	+.08	8	+1.46	+.14	7	....	.....	..
8.16	+.37	+.06	7	+.59	+.14	9	+1.12	+.23	9	+1.58	+.32	9	....	.....	..

Fig. 12 is a graphic representation of the same data showing clearly the systematic differences. In the two brighter groups but slight differences appear. In group 3, mean magnitude 7.78, the agreement is good for the white stars, but for the colored stars the differences become evident, though not large. The faintest group contains only colored stars, on account of the method of selection of the observing list. In this group the differences increase rapidly with the color. It should be noted, however, that the exposures were not designed to give stars as faint as the last two divisions of the group, so that their magnitudes are somewhat uncertain. In response to an inquiry Professor Schwarzschild

kindly informed me that the faintest stars (those that would give a difference with a negative sign) were not measured at Göttingen

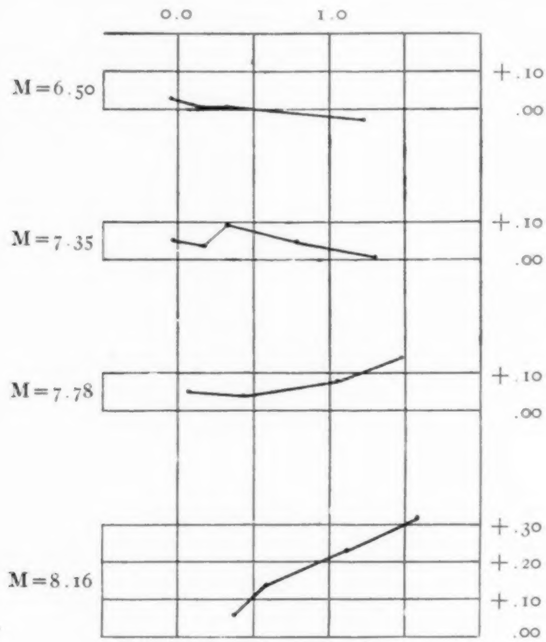


FIG. 12.—Comparison with Schwarzschild

on account of their faintness, thus leaving a systematic difference from the method of measurement, rather than from any peculiarity of the stars themselves.

COMPARISON WITH MÜLLER AND KEMPF'S VISUAL MAGNITUDES

The 598 stars in common with *Potsdam*, **17** were arranged in eight groups of four or five divisions each. Table VII gives the differences in the sense, P minus M&K.

The curves in Fig. 13, representing the above data show a very satisfactory agreement, indicating not only that the scale is nearly the same as that of Müller and Kempf, but that the sensitiveness to color of the plate behind the filter agrees closely with the eyes of the Potsdam observers. Here, also, the first and last exposures

are entitled to less weight, as they include extremes in brightness. Omitting the extreme groups, the difference in color-perception

TABLE VII

Mean Mag.	Division 1			Division 2			Division 3			Division 4			Division 5		
	Color	Δ Mag.	No.	Color	Δ Mag.	No.	Color	Δ Mag.	No.	Color	Δ Mag.	No.	Color	Δ Mag.	No.
5.47	−.23	+.09	11	+.12	+.08	12	+ .56	+.05	12	+1.16	+.09	12	+1.64	.00	12
6.30	−.03	+.01	13	+.25	−.05	13	+ .73	−.02	11	+1.11	−.05	12	+1.61	−.04	14
6.75	−.04	+.01	20	+.17	+.01	20	+ .43	−.03	16	+1.03	−.01	19	+1.50	−.01	24
7.11	+.07	−.02	20	+.41	−.02	15	+ .66	.00	16	+1.26	.00	20	+1.66	.00	14
7.38	−.01	+.01	20	+.24	.00	25	+ .57	.00	20	+1.05	.00	26	+1.44	−.02	21
7.63	−.02	+.03	15	+.23	+.02	16	+ .46	.00	15	+ .95	−.04	16	+1.30	−.03	14
7.85	+.03	+.02	15	+.21	−.04	13	+ .44	+.03	13	+ .75	+.02	10	+1.26	−.08	9
8.21	−.06	+.09	10	+.27	+.07	12	+ .52	+.01	10	+ .90	−.05	12	....	.....	..

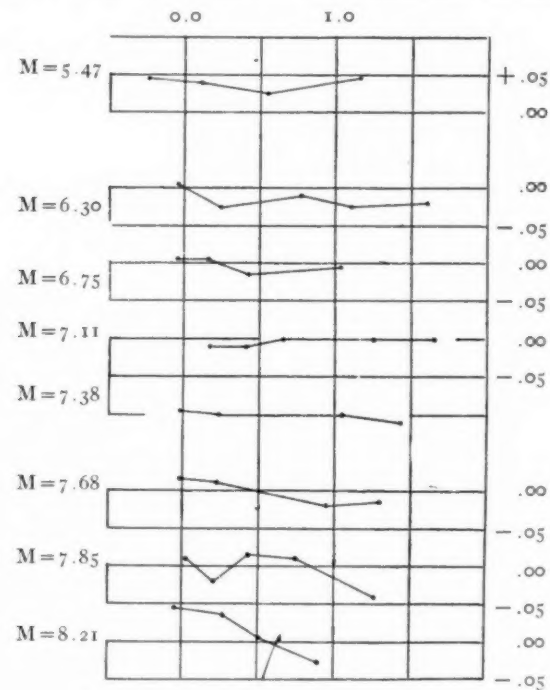


FIG. 13.—Comparison with Müller and Kempf

introduces a difference in magnitude of 2 per cent. Including the extremes, it becomes 3 per cent. The sign of the difference

is such that if the eyes of the Potsdam observers are taken as the standard, the color-correction of the plate and filter is overdone, so that the faint colored stars appear too bright on the plate.

COMPARISON WITH MÜLLER AND KEMPF'S COLOR-ESTIMATES

The same grouping of stars used for Müller and Kempf's visual magnitudes serves for the comparison with their estimates of color. In Fig. 14 the Potsdam color-steps are platted as uniform increments in abscissas, while the ordinates are values of the color-index. The points representing the mean values for each magnitude-group are connected by lines, with the mean magnitude at its upper end. For the white stars at the lower left, the lines of the difference groups lie close together, but for the colored stars at the upper right, the lines diverge. The line representing the brightest group, mean magnitude 5.47, lies well to the right of the rest. As the stars become fainter the lines move to the left. This is a demonstration of the law of physiological optics, that the apparent maximum of intensity shifts toward the shorter wave-lengths as the light is reduced in brightness; in other words, a given color is less evident in a faint star than in a bright one.

The massing of the platted points, near the GW (yellowish-white) line, seems to indicate a preference of the Potsdam observers for that color.

COMPARISON WITH THE HARVARD VISUAL MAGNITUDES

The tables given by Müller and Kempf[1] furnish the best means of comparing the magnitudes obtained with the "visual-luminosity" filter with the visual magnitudes found at Harvard. Applying these tabular differences to the quantities given in Table VII, we have the values in Table VIII; where each magnitude group is divided according to values of the color-index into four or five divisions.

It follows from the above differences, which are in the sense Parkhurst minus Harvard: first, for the white stars the scale is very nearly the same; second, the colored stars are made a little

[1] *Potsdam Photometric Durchmusterung*, p. xxxiv.

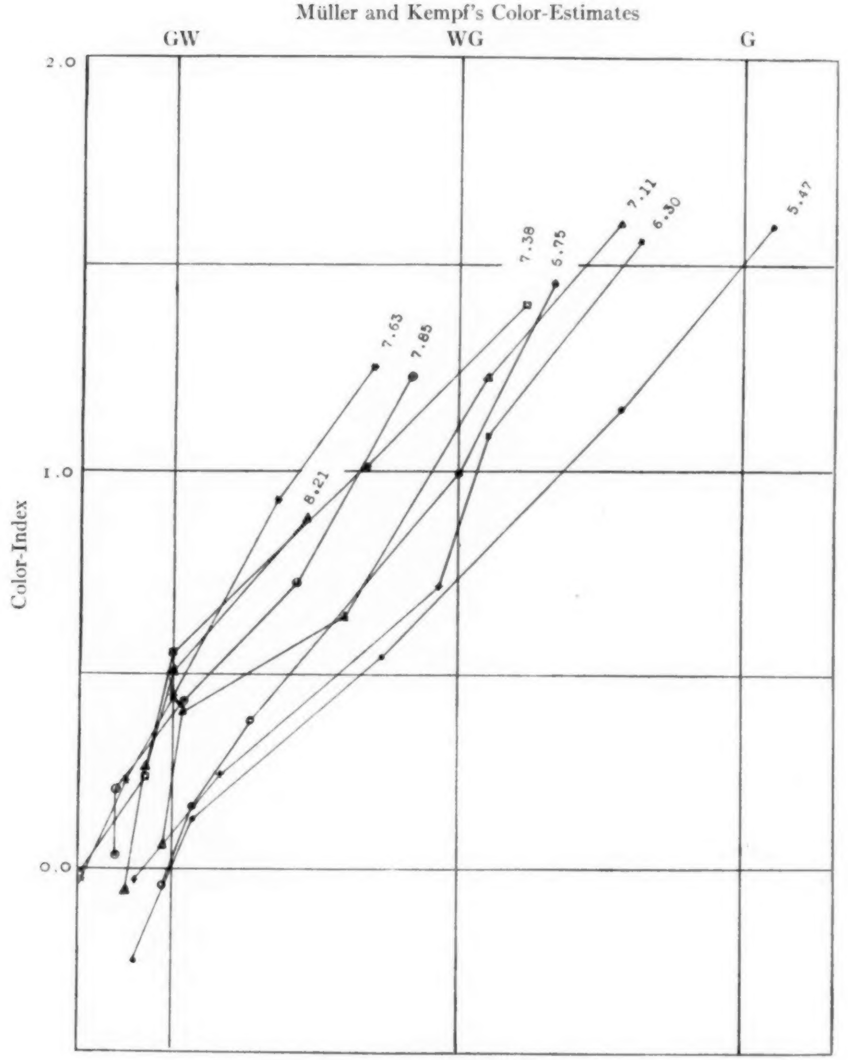


FIG. 14.—Comparison of color-index with Müller and Kempf's color-estimates

brighter than in the Harvard catalogues, for the extremes the differences being about one-quarter of a magnitude.

TABLE VIII

COMPARISON WITH THE HARVARD VISUAL MAGNITUDES

Mean Mag.	Division 1		Division 2		Division 3		Division 4		Division 5	
	Color	ΔMag.	Color	ΔMag.	Color	ΔMag.	Color	ΔMag.	Color	ΔMag.
5.47......	−0.23	+.08	+0.12	+.08	+0.56	−.09	+1.16	−.03	+1.64	−.25
6.30......	−0.03	+.02	+0.25	−.12	+0.73	−.15	+1.11	−.24	+1.61	−.35
6.75......	−0.04	.00	+0.17	.00	+0.43	−.10	+1.03	−.20	+1.50	−.24
7.11......	+0.07	−.02	+0.41	−.10	+0.66	−.14	+1.26	−.20	+1.66	−.28
7.38......	−0.01	+.06	+0.24	.00	+0.57	.00	+1.05	−.13	+1.66	−.24
7.63......	−0.02	+.06	+0.23	+.04	+0.46	+.01	+0.95	−.10	+1.30	−.16
7.85......	+0.03	+.05	+0.21	−.03	+0.44	+.04	+0.75	−.04	+1.26	−.24
8.21......	−0.06	+.13	+0.27	+.10	+0.52	+.03	+0.90	−.11	......	.....

COMPARISON WITH GREENWICH MAGNITUDES OF STARS NEAR THE NORTH POLE

The list given in *Monthly Notices*, **72**, 695, June 1912, contains 42 stars in common with this zone. The magnitudes there given are based directly on the Harvard Polar Sequence. The stars were arranged in seven groups of six stars each, and showed the following residuals, in the sense P−G.

Group	Mean Mag.	Δ Mag.
1....................	5.43	−0.02
2....................	6.57	+0.06
3....................	6.79	+0.15
4....................	7.19	+0.20
5....................	7.41	+0.28
6....................	7.63	+0.25
7....................	7.95	+0.18

The stars were too few in number to admit of forming subgroups according to color-index, but the residuals were evidently greater for stars of more intense color. This is clearly shown by averaging the values of the color-index for the stars whose residuals are respectively below and above the mean in each group.

Group	Average Color-Index	
	Below Mean	Above Mean
1	−0.17	+0.17
2	+0.14	+0.72
3	+0.32	+0.27
4	+0.18	+0.26
5	+0.78	+1.24
6	+0.56	+1.05
7	+0.85	+1.66
Mean	+0.43	+0.77

In all the groups except the third, a large residual corresponds to a large value of the color-index. To determine the numerical value of the difference due to color, a greater number of stars would be required.

Excluding *Polaris*, which is too bright for this work, there are eleven stars of the Harvard Polar Sequence in the above list. They range from 4.47 to 7.73 in magnitude. The above tables therefore furnish a rough comparison with the Harvard magnitude scale. A careful comparison is reserved for a subsequent investigation.

DISTRIBUTION OF SPECTRAL CLASSES

The spectra of 492 stars were clear enough on the plates to admit of good classification. Just half of them were included in the limits B2 to F0, and half between F1 and M. The numbers in the separate classes are as follows:

Class	No.	Percentage
B2 to B9	30	6
A0 to A9	196	40
F0 to F9	78	16
G0 to G9	112	23
K0 to K9	70	14
M	5	1
N	1	..

It would seem to be unnecessary to go into details in regard to the distribution with respect to the Galaxy. The difference in the number of white standard stars per field was, however, very notice-

able. At the nearest approach to the Galaxy, 10° at right ascension 1 hour, the number of white stars per field reached 10 or 12; but at the greatest distance, 44° at 13 hours, the number of standards fell to 2 or 3.

SUMMARY

The main characteristics of the present work are as follows:

1. The determination, with the same instrument, of the three photometric elements, photographic magnitude, visual magnitude, and spectrum (and therefore the relation between spectrum and color-index).

2. The use of extra-focal images and an absolute scale for the photographic magnitudes.

3. The use of color-sensitive plates and a filter especially designed to give, with the particular plates used, magnitudes on the visual scale.

4. The statement of the spectral intensity curve of each kind of plate used, in the condition used; thus defining the meaning of the results obtained.

5. The correction of the photographic magnitudes for the color-curve of the lens.

6. The comparison with visual and photographic magnitudes found at Harvard, Potsdam, and Greenwich, showing systematic differences but a reasonable degree of agreement.

ACKNOWLEDGMENTS

My thanks are especially due to Dr. Anne S. Young, professor of astronomy in Mt. Holyoke College, who measured and reduced in duplicate all the focal plates in the 75° zone, determined part of the systematic corrections, and furnished expert advice and aid in many questions arising during the work; to Miss Eudora Magill, formerly of the computing staff of Columbia College Observatory, for careful and accurate work in measuring and reducing focal plates and arranging the catalogue; to the Trustees of the Rumford Fund of the American Academy, for aid in purchasing the Hartmann "Mikrophotometer"; and to the Trustees of the Bache Fund

of the National Academy, for a grant to pay for assistance in measuring and reducing plates.

Acknowledgments have already been partially made of the important part taken by Professor F. C. Jordan in devising and perfecting the "absolute scale," which forms the basis of so much of the work.

YERKES OBSERVATORY
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THE ABSORPTION SPECTRUM OF TELLURIUM VAPOR AND THE EFFECT OF HIGH TEMPERATURE UPON IT

By E. J. EVANS

The absorption spectrum of tellurium vapor was first investigated by Gernez,[1] but, as far as the present author is aware, the effect of high temperatures on the spectrum has not been studied. Gernez heated tellurium in an atmosphere of carbon dioxide in a hard glass tube, and obtained a banded absorption spectrum extending from yellow to violet. The emission spectrum of the vapor at high temperatures was studied by Paterno and Mazzucchelli.[2] They found that the yellow tellurium vapor became green with increase of temperature, and finally blue at the highest temperatures employed. The emission spectrum showed the presence of a large number of bands.

The present research is a continuation of previous work carried out by Antonoff and the author[3] on the absorption spectrum of selenium vapor. The vapor-densities of several substances including selenium and tellurium were investigated by Deville and Troost.[4] In the case of selenium they found a diminution of vapor-density with increase of temperature, pointing to the conclusion that complex molecules were being dissociated into Se_2. They were unable in the case of tellurium to observe any change in the vapor-density with change of temperature, but this they regarded as being due to impure material. According to Deville and Troost the vapor-density of tellurium at temperatures above the boiling-point (1390° C.) is 130 ($H=1$). The molecule of tellurium at these high temperatures can be represented by Te_2. From analogy with sulphur and selenium there is reason to believe that there are complex molecules present in tellurium vapor especially at low temperatures. The boiling-point of tellurium in

[1] *Comptes rendus*, **74**, 1190, 1872.

[2] *Accad. Lincei Atto*, **17**, 428, 1908.

[3] *Astrophysical Journal*, **34**, 277, 1911.

[4] *Comptes rendus*, **56**, 891, 1863.

a cathode vacuum is about 500° C. according to Krafft,[1] and it would be interesting to determine the vapor-density at this comparatively low temperature. The present paper is mainly concerned with the changes produced in the absorption spectrum of the vapor by increase of temperature. These changes are probably connected with the dissociation of complex molecules into simpler ones. At present it is difficult to give a precise meaning to the results obtained, because our knowledge of the vapor-density of tellurium at different temperatures and pressures is so meager.

EXPERIMENTAL ARRANGEMENT

The experimental arrangement was similar to the one previously employed in the investigation of the absorption spectrum of iodine,[2] and consequently only a short description is necessary. The tellurium vapor was heated in a quartz tube placed inside an electric furnace, whose temperature was measured by a Pt Pt–Rh thermocouple. The quartz tube was also provided with a side tube of 3 or 4 mm bore in which the tellurium was placed, and after the apparatus had been evacuated to a pressure of $\frac{1}{10}$ mm or less, the side tube was sealed off in the oxyhydrogen flame. The pressure of the vapor could be varied by sending currents of different magnitudes through a nickel wire wound round the side tube. The temperature of the coldest part of the side tube was determined, as the vapor pressure inside the apparatus depended upon it. When the furnace and side tube had been adjusted to their respective temperatures, the light from the positive pole of the carbon arc was passed through the vapor and focused on the slit of a concave grating of 1 meter radius and 15,000 lines to the inch. The absorption spectrum was usually photographed with Wratten and Wainwright's panchromatic films. Two different specimens of tellurium were employed, and the spectra obtained under similar conditions were found to be identical. One of the specimens was known to have been carefully prepared from the pure oxide, but the other was suspected of containing a small quantity of an impurity (probably copper). Since the maximum temperature of the side

[1] *Berichte der Deutschen chemischen Gesellschaft*, **36**, 4344, 1903.

[2] *Astrophysical Journal*, **32**, 1, 1910.

tube in the present experiments was about 700° C., any metallic impurities such as copper—which has a high boiling-point—would not be present in sufficient quantity to produce an absorption spectrum. A careful examination of the photographs also showed that the tellurium did not contain an appreciable quantity of selenium as an impurity. This could be easily tested, as selenium is much more volatile than tellurium, and absorbs strongly in the violet and ultra-violet. In all experiments with tellurium vapor the quartz tube was evacuated to a pressure of $\frac{1}{10}$ mm or less, for tellurium combines readily with oxygen, forming a white oxide. This oxide melts at red heat, forming a yellow liquid, and at a still higher temperature boils. It was found that if the oxide was heated in a quartz tube for some time at a high temperature, a white incrustation was formed on the clear ends of the tube, preventing the transmission of the light from the electric arc to the slit of the grating spectroscope. The quartz tubes became porous after long-continued exposures to high temperatures, and consequently such tubes had to be frequently replaced by new ones. The absorption spectra were examined for the region λ 2400 to λ 7000, and for temperatures varying from about 500° C. to 1350° C. In one series of experiments the pressure was kept constant and the temperature varied, while in another series a known weight of tellurium was placed in a quartz tube and the absorption spectrum photographed at different temperatures.

EXPERIMENTAL RESULTS

THE DIOXIDE SPECTRUM

As tellurium dioxide might be present as an impurity in the experiments on the absorption spectrum of tellurium, a brief examination of the absorption spectrum of the former was made. A few grams of the dioxide were prepared by dissolving tellurium in nitric acid and evaporating to dryness. About 0.004 gm of the oxide was then sealed in a quartz tube and the absorption spectrum photographed at temperatures ranging from 700° C. to 1050° C. In the first series of experiments glass lenses were employed, and the absorption spectrum could be investigated from λ 3500 to λ 7000. The light from the positive pole of the arc after passing

through the vapor contained in the tube at a temperature of 700° C. gave a continuous spectrum extending from λ 3500 to λ 7000, and no definite absorption bands could be detected. The temperature of the vapor was raised to 800° C., and the spectrum again photographed. The spectrum extended from λ 7000 to λ 3800, and there was complete absorption between the latter wave-length and λ 3500. The photograph also showed the presence of faint, ill-defined bands between λ 3880 and λ 4100. The temperature of the vapor was finally raised to 1050° C., and the spectrum of the transmitted light photographed. A continuous spectrum was obtained between λ 7000 and λ 4100, but no definite absorption bands could be detected in that region. There was complete absorption between λ 4100 and λ 3500. A number of photographs were also taken at various temperatures of the vapor in order to study the ultra-violet part of the spectrum. For this purpose the glass lenses were replaced by those of quartz. At 600° C. or 700° C. a continuous spectrum extending from λ 3885 to λ 2800 was obtained, and no absorption bands appeared on the photographs. Bands were, however, obtained when the temperature of the vapor was higher (900° C.). These bands, which are found between λ 3100 and λ 3400 are not due to tellurium vapor, although they may possibly be due to some impurity in the oxide. Experiments on the absorption spectrum of the oxide at high temperatures were rendered difficult by its action on the clear ends of the quartz tube.

GENERAL DESCRIPTION OF THE ABSORPTION SPECTRUM OF TELLURIUM VAPOR

A small quantity of tellurium was placed in a quartz tube, which was evacuated to a pressure of $\frac{1}{10}$ mm and the absorption spectrum studied for temperatures of the furnace ranging from about 500° C. to 1000° C. In the first series of experiments glass lenses were employed and the region of the spectrum that could be photographed was comprised between λ 3500 and λ 7000. When the temperature was about 500° C. faint absorption bands extending from λ 3900 to λ 4200 were obtained. No bands could be detected in the region of higher wave-length or in the region between λ 3900 and λ 3500. The temperature of the furnace was then raised

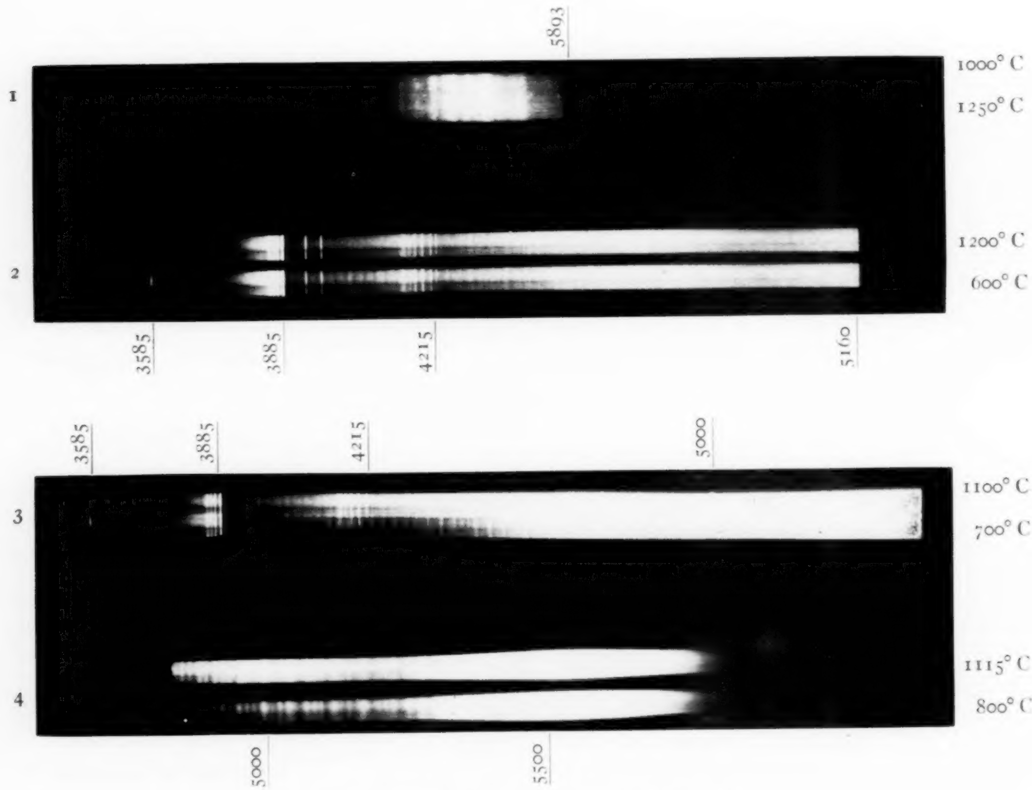
to 700° C., and the absorption bands in the above region (λ 3900–λ 4200) increased in intensity and extended toward the red end of the spectrum. The absorption bands now extended from λ 3900 to λ 5200, and the photographs also showed the presence of continuous absorption between λ 3900 and λ 3500. The temperature was then successively raised to 800° C., 900° C., and 1000° C., and the absorption spectra, photographed, gave the following results. At 800° C. the general absorption in the ultra-violet increased, complete absorption was obtained between λ 3885 and λ 4400, and an increase in the intensity of the absorption bands between λ 4400 and λ 5400. When the temperature was 900° C. complete absorption was obtained in the region λ 5000–λ 3500, and the absorption bands increased in intensity toward the red end of the spectrum. In this case they could be recognized as far as λ 5700. At 1000° C. absorption bands could be detected as far as λ 6100, and this was the limit attained with the small amount of substance used in the present work. The glass lenses were now replaced by others made of quartz, so that the absorption spectrum could be studied in the ultra-violet (λ 4000–λ 2500).

When the temperature of the furnace was 700° C. a continuous spectrum extending from λ 3885 to λ 2500 and showing no absorption bands was obtained. There was, however, in this region a general absorption which decreased in intensity with diminishing wave-length. At a higher temperature (900° C.) there was complete absorption from λ 4000 to λ 3450, and no absorption bands were visible in the region λ 3450–λ 2600. Finally the temperature of the furnace was raised to 1100° C., and the absorption spectrum photographed in the visible part as well as in the ultra-violet. Absorption bands were obtained extending from λ 6000 to λ 5300, complete absorption from λ 5300 to λ 3400, and a continuous spectrum showing no absorption bands extending from λ 3400 to λ 2600.

EFFECT OF TEMPERATURE ON THE ABSORPTION SPECTRUM CONSTANT MASS OF VAPOR IN TUBE

About 0.002 gm of tellurium was introduced into a quartz tube, which was evacuated to a pressure of a fraction of a millimeter and sealed. The tube was placed inside a carbon-tube furnace, and

PLATE X



Absorption Spectra of Tellurium Vapor

the absorption spectrum was examined visually at temperatures ranging from 1000° C. to 1200° C. The initial temperature was taken as high as 1000° C. to insure that all the tellurium in the tube was in a state of vapor. The visual observations showed scarcely any difference in the appearance of the absorption spectrum as the temperature was raised, and this was confirmed by the careful examination of photographs taken at 1000° C., 1250° C., and 1350° C. All the photographs were taken with the same exposure and showed the presence of absorption bands extending from λ 6000 to λ 5390, and increasing in intensity with diminishing wave-length. There was in each case complete absorption of all wave-lengths from λ 5390 to λ 3500—the limit of photography with the glass lenses employed. The absorption bands at 1000° C. and 1250° C. are shown in Plate X, Fig. 1. The two spectra overlap slightly.

CONSTANT-PRESSURE EXPERIMENTS

In these experiments the vapor pressure of the tellurium was kept constant while the absorption spectrum was photographed at a low and a high temperature. The actual value of the pressure was not determined, but the temperature of the coldest part of the apparatus was always observed. The vapor pressure was varied over a range corresponding to a change of temperature from about 500° C. to 750° C. The absorption bands in the region λ 3900–λ 4200 could be detected when the temperature of the coldest part of the apparatus was only 20° or 30° C. above the melting-point (450° C.) of tellurium, and as the normal boiling-point of tellurium is 1390° C. the vapor pressure required to detect the bands must be small. The experimental results for a few values of the vapor pressure will now be given.

Experiment I.—In this experiment the temperature of the coldest part of the apparatus was 540° C., and two photographs were taken with the same exposure, one at a temperature of 600° C. and the other at 1200° C. The absorption bands which could be easily detected at the low temperature were much less intense at the high temperature. Their diminution in intensity is shown in Plate X, Fig. 2. The carbon arc lines are also prominent in the photograph.

Experiment II.—The vapor pressure was greater than in the first experiment and corresponded with a temperature of 580° C. Two photographs were taken, one at a temperature of 700° C. and the other at 1100° C. The absorption bands in the region λ 3900 to λ 5000 were much reduced in intensity at the higher temperature. The result of this experiment is shown in Plate X, Fig. 3.

Experiment III.—The vapor pressure was again increased and corresponded with a temperature of 700° C. The absorption spectrum was photographed at 830° C. and 1100° C. At the lower temperature absorption bands were obtained in the region λ 5300 to λ 4500 and the light from the positive pole of the arc comprised between λ 4500 and λ 3500 was completely absorbed. The absorption bands at the higher temperature were not as intense as at the lower temperature, but they extended farther toward the violet (to λ 4300) before complete absorption set in.

Experiment IV.—In this case the temperature of the coldest part of the apparatus was not measured directly, but from the current passing through the nickel wire it was estimated to be about 670° C. Two photographs were taken at 800° C. and 1115° C., but the effective exposure at the higher temperature was slightly greater than at the lower temperature. The photograph taken at 800° C. showed the presence of absorption bands extending from λ 5300 to λ 4500; the intensity of the absorption increasing with diminishing wave-length. As in experiment III, there was complete absorption between λ 4500 and λ 3500. At the higher temperature the bands extended from λ 5300 to λ 3900, and the difference in the spectra was too great to be explained by the slight difference in exposure. The region comprised between λ 4800 and λ 6000 is shown in Plate X, Fig. 4.

EXPERIMENTS IN THE ULTRA-VIOLET

Although previous experiments indicated that tellurium vapor did not give any absorption bands in the ultra-violet, it was considered advisable to perform a few experiments at a high and a low temperature when the vapor pressure was kept constant. The temperature of the coldest part of the apparatus was kept constant at 530° C. and the ultra-violet portion of the spectrum photo-

graphed when the temperature of the vapor was 600° C. and also 1000° C. In each case no absorption bands could be detected in the region λ 3885–λ 2800. The same result was obtained when the vapor pressure was higher (corresponding to a temperature of 680° C.).

DISCUSSION OF RESULTS

The experimental results show that tellurium vapor gives a banded absorption extending from λ 6100 to λ 3900, and also a general selective absorption. It is possible that the absorption bands may extend still farther toward the red if greater vapor pressures are employed. Previous work on the absorption spectrum of selenium vapor shows that the bands extend over the region λ 5300 to λ 3200, and according to Graham[1] the absorption bands of sulphur vapor in the region λ 4775–λ 3985 are due to S_8 molecules and bands in the ultra-violet (λ 3415–λ 2600) are due to S_2 molecules. A comparison of the absorption spectra of these three substances, which belong to the same group in the periodic classification of the elements, shows that the banded absorption spectrum is shifted toward the red with increasing atomic weight. If the absorption spectra of tellurium dioxide and selenium dioxide are compared, it is found that while the latter gives well-defined bands in the violet and blue, the former does not give any bands in the visible spectrum. One would expect from analogy between the behavior of tellurium and selenium that tellurium dioxide would give bands in the green and yellow. The absorption spectra of tellurium and selenium, both in their appearance and behavior with change of temperature and pressure, show many points of resemblance. When tellurium is heated in a quartz tube absorption bands first make their appearance at λ 3900, and as the pressure increases the bands extend toward the red end of the spectrum. Another consequence of the increased pressure is that the region in which the bands appear at a lower pressure is completely absorbed at the higher pressure. The same general result is obtained in the case of selenium, the only difference being that the bands first make their appearance at λ 3200. The resemblance is again evident when the effect of temperature on the absorption spectra of the two vapors

[1] *Proceedings Royal Society*, A, **84**, 311, 1910.

at constant pressure is considered. When the pressure of the tellurium vapor is small, absorption bands are obtained in the region λ 3900–λ 4500, and the intensities of these bands diminish with increase of temperature until they almost disappear at 1200° C. A similar result is obtained with selenium vapor; the absorption bands in the region λ 3200–λ 3800 diminish in intensity with increase of temperature and almost disappear at 1200° C. The disappearance of the absorption bands with increase of temperature can be explained on the hypothesis that complex molecules of tellurium are dissociating into simpler ones, and that the presence of the more complex molecules is necessary for the production of these bands. At present the nature of the complex molecule responsible for the absorption bands in the region λ 3900–λ 4500 is unknown; also, the number of bands belonging to a particular complex molecule is difficult to determine.

In experiment II (constant pressure) the bands between λ 3900 and λ 5000 are diminished in intensity at the higher temperature, and possibly all the bands between λ 3900 and λ 5300 would disappear at temperatures above 1500° C. It is interesting to consider experiment IV (constant pressure), paying special attention to the region λ 3900–λ 4500. At 800° C. there is complete absorption in that region, but at 1115° C. distinct absorption bands are obtained. The pressure of the vapor in this experiment is large compared with that employed in experiment I, and consequently the bands are not very faint even at 1100° C. It would require a much higher temperature to bring about the complete disappearance of the bands. The appearance of the high-temperature spectrum in the above region suggests that a general selective absorption is superimposed on the band absorption, and it is interesting to inquire into the probable origin of this general absorption. Koenigsberger,[1] as the result of investigations on the absorption spectra of several substances, comes to the conclusion that continuous selective absorption is due to the normal state of gas molecules, and that band spectrum absorption is due to an intermediary non-stationary condition. In the absorption spectrum of tellurium the two types of absorption are present, but the hypothesis cannot be proved or

[1] *Astrophysical Journal*, **35**, 139, 1912.

disproved by means of the experimental results. The band absorption is undoubtedly connected with dissociation, but from the results obtained, it is impossible to say whether the bands are produced by the molecules in their non-stationary condition or by molecules in their natural state. As to the origin of the general absorption several hypotheses are possible; the continuous absorption may be due to the more complex molecules in their normal state or due to the diatomic molecule. Also both types of molecules may produce general selective absorption in the same region of the spectrum. Further researches on the variation of the intensity of the general absorption with increase of temperature, when the vapor-density of tellurium over a wide range of temperature and pressure is known, may throw light on this question. In conclusion, the experiments on a constant mass of tellurium sealed in a quartz tube will be considered. The photographs taken at temperatures above 1000° C. seem to indicate that all (0.002 gm) the tellurium is in the state of vapor, and consequently the pressure of the vapor at any temperature above 1000° C. can be estimated by means of the equation $pv=R\theta$. On the assumption that all the molecules are diatomic the pressure at 1300° C. is found to be about 5 cm of mercury. The photographs of the absorption spectrum at 1000° C., 1200 °C., and 1350° C. show the presence of bands in the region λ 5300–λ 6000, and complete general absorption between λ 5300 and λ 3500. Further, the intensities of the absorption bands are not affected by a change of temperature from 1000° C. to 1350° C. It is unfortunate, however, that quartz tubes become porous and opaque at higher temperatures, making it impossible to obtain information which would have an important bearing on the question of the origin of the bands. The nature of the molecule responsible for these bands is left undetermined, but it is interesting to note that these bands are present when the vapor is at a high temperature, and probably almost completely diatomic.

SUMMARY OF RESULTS

1. The bands in the absorption spectrum of tellurium extend from λ 3900 to about λ 6000. In addition to a band spectrum the vapor gives a general selective absorption. The absorption

spectrum of tellurium is similar to that of selenium, but compared with the spectrum of the latter is displaced toward the red.

2. For small pressures absorption bands are first photographed in the extreme violet (λ 3900), and as the pressure increases, absorption bands are also photographed in regions of greater wave-length. At pressures sufficient to show the presence of bands in the region λ 5300–λ 6000, there is complete absorption in the violet and blue.

3. When the pressure of the vapor is low, the absorption bands in the region λ 3900–λ 4500 diminish in intensity with increase of temperature until they almost disappear at 1200° C. This result may be explained on the hypothesis that the absorption bands are due to complex molecules, which are present in tellurium vapor at low temperatures. These molecules are completely dissociated at high temperatures and hence the absorption bands are not visible.

4. The absorption spectra of a constant mass (0.002 gm) of tellurium vapor at 1000° C. and 1350° C. do not show any difference. Both spectra show the presence of bands between λ 5300 and λ 6000 and a continuous absorption from λ 5300 to λ 3500. The intensities of the bands are not affected by a change of temperature from 1000° C. to 1350° C. From the experimental results it is impossible to determine whether these bands are due to diatomic molecules or more complex ones.

I take this opportunity of thanking Professor Rutherford for the interest he has taken in the work and for placing the necessary facilities at my disposal.

Physics Laboratory
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May 1912

ON DARKENING AT THE LIMB IN ECLIPSING VARIABLES. I

By HENRY NORRIS RUSSELL AND HARLOW SHAPLEY

1. *Solution for total eclipses of completely darkened stars.*—In the problem of determining the orbital elements of an eclipsing binary from the light-curve, the nearly universal custom has been to assume that the stars are without appreciable absorbing atmospheres and that the stellar disks, therefore, are uniformly bright from the center to the limb. The present accuracy of photometric observations has been considered insufficient to justify introducing into the theory the complications which a contrary hypothesis would admit; moreover, the few reliable orbits so far computed on the assumption of uniform disks have represented the observations very satisfactorily. The extreme unlikelihood, however, of uniform disks has been generally recognized, the analogy of the sun being definite evidence in the case of stars of advanced spectral type, and attempts have been made by Rödiger[1] and Blažko[2] to determine the elements of *β Persei* and *U Cephei*, respectively, on the assumption of a differential darkening on the stellar disks equal to that of the sun. These investigations were not prompted by the discrepancy between observations and the uniform-disk theory but rather by the reasonableness of the opposing assumption.

The general problem of darkening at the limb has already been discussed by one of us in his second article on the determination of the orbital elements of eclipsing variable stars,[3] and it is the purpose of the present paper to carry out the investigation there outlined, constructing tables to facilitate the solution of the orbits of darkened stars and discussing the errors which the occurrence of darkening at the limb may produce in elements computed on the

[1] *Untersuchungen über das Doppelsternsystems Algol*, Königsberg, 1902; cf. Stebbins, *Astrophysical Journal*, **32**, 200 and 207, 1910.

[2] "Sur les étoiles variable du type Algol," *Annales de l'observatoire astronomique de Moscou*, Série II, **5**, No. 9, 1911.

[3] Russell, *Astrophysical Journal*, **36**, 69 ff., 1912.

assumption that it does not exist. The notation and fundamental equations are those of the papers just mentioned.

If we take

$$J=J_0\,(1-x+x\cos i) \tag{1}$$

as a sufficiently general law of variation of the surface brightness, J, on a star's disk, where i is the angle between the line of sight and the normal, we may consider x the darkening coefficient. For $x=0$ the surface is uniformly illuminated; for $x=1$ the star is completely darkened at the limb. Denoting by α the fraction of the light of the eclipsed star that is intercepted by the companion (in the case of a uniform disk α is equivalent to the fraction of the area obscured), we see that this quantity depends on the darkening coefficient, the radii of the component stars, r_1 and r_2, and the apparent distance of centers, δ—or, in fact, on x and the ratios of the last three quantities. For fixed values of the ratios $\frac{r_2}{r_1}=k$ and $\frac{\delta}{r_1}$, α becomes a linear function of x, which, for $x=0$, is a familiar function of the remaining variables, $f\left(k, \frac{\delta}{r_1}\right)$, and for $x=1$ may be written $f^1\left(k, \frac{\delta}{r_1}\right)$, so that in general

$$\alpha^x=(1-x)f\left(k, \frac{\delta}{r_1}\right)+xf^1\left(k, \frac{\delta}{r_1}\right)=f^x\left(k, \frac{\delta}{r_1}\right). \tag{2}$$

Now the function $f\left(k, \frac{\delta}{r_1}\right)$, which is a fundamental relation in the uniform-disk theory, has been computed in the previous paper and its inverse tabulated.[1] If we compute $f^1\left(k, \frac{\delta}{r_1}\right)$, which is a similar but much more complicated function, for a system of stars darkened to zero at the limb, we can then, from (2), by interpolation readily determine $\alpha^x=f^x\left(k, \frac{\delta}{r_1}\right)$ for any desired value of the darkening coefficient.

The computation by mechanical quadratures of α^1 for several different values of the geometrical variables, k and $\frac{\delta}{r_1}$, will be facilitated if we replace δ by r_1+r_2p, where p, a function of k and α^1,

[1] *Astrophysical Journal*, **35**, 333, 1912.

is the apparent distance, measured along the line of centers in units of the radius of the smaller star, from the circumference of the larger star to the center of the smaller one; so that we may write

$$a^{1}=F^{1}(k,\ p), \tag{3}$$

and proceed to determine the amounts of light lost, for given values of k, as p varies from $+1$ at the beginning of eclipse to -1 at the second contact (we are considering here only total eclipses).

For the purpose of the quadrature a standard disc of 16 inches diameter was first constructed, representing the apparent surface of the eclipsed star, and was divided radially into 64 equal sectors. These were then cut by 24 circles concentric with the disk, thus dividing it into 1600 parts, the lengths of the radii of the successive circles being so determined that an equal amount of light is received from each part. Since the total light received from the inside of a concentric circle of radius sin θ, drawn upon the disk completely darkened at the limb, whose radius is taken as unity, is

$$L=\int_0^\theta J d\sigma$$

where $J=C\cos\theta$ is the apparent surface brightness of the projected element of surface, $d\sigma=2\pi\sin\theta\cos\theta d\theta$, that is, since

$$L=\int_0^\theta 2\pi C\cos^2\theta\sin\theta d\theta=\tfrac{2}{3}\pi C(1-\cos^3\theta)\ , \tag{4}$$

we see that the light varies directly with the cube of the cosine of the angle subtended at the center of the star by radii drawn on the apparent disk. Therefore, by computing the radii of the concentric circles, $r=\sin\theta$, from 24 equidistant values of $\cos^3\theta$(0.04, 0.08, 0.12, ... 0.96), 25 circular segments are obtained from which equal amounts of light are received. Upon this completed diagram of 1600 equally potential divisions was superposed a sheet of transparent paper on which circles of radius $\frac{1}{k}$ were drawn for k equal to 1.0, 0.8, 0.6, 0.4, and 0.2, that is, of 16 inches, 20 inches, $26\frac{2}{3}$ inches, etc., diameter. These circles, representing the eclipsing star with different values of the radius, were superposed so that the circumference cut the radius of the diagram normally at the distances p from the center. By simply counting and tabulating

the divisions and fractions of divisions included between the intersecting circumferences, the quadrature for a^1 was rapidly effected and checked by difference curves. From this tabulated material, curves, represented by equation (3), were drawn for fixed values of k. For $k=0$ the relation between a^1 and p is algebraic,

$$4a^1=(1+p)^2(2-p),$$

and the smoothness of the differences of the other curves from the plot of this equation afforded a further check on the quadrature.

Having assigned definite values to k, it is possible to invert the a^1-function just determined and read off from the curves p as a function of a^1 and k. This has been done to form Table Ix of this paper, which is analogous in every way to Table I of the former article.[1] The two tables are fundamental for the uniform disk and completely darkened disk hypotheses, respectively, and interpolation between them, or rather between the inverted functions which precede them, forms a ready basis for the investigation of intermediate degrees of darkening.

From the relation $\delta=r_1+r_2p(k, a^1)$, we have

$$\frac{\delta}{r_1}=1+kp(k, a^1), \tag{5}$$

and since, geometrically,

$$\delta^2=\sin^2\theta+\cos^2 i\cos^2\theta, \tag{6}$$

where i is the inclination of the orbit to the tangent plane, $\theta=\frac{2\pi t}{P}$ is the orbital angle of the eclipsing star counted from minimum, and t is the corresponding time expressed in the same units as the period, P, we obtain the fundamental relation

$$r_1^2\{1+kp(k, a^1)\}^2=\cos^2 i+\sin^2 i\sin^2\theta, \tag{7}$$

and, further, by a simple reduction,

$$\frac{\{1+kp(k, a_1^1)\}^2-\{1+kp(k, a_2^1)\}^2}{\{1+kp(k, a_2^1)\}^2-\{1+kp(k, a_3^1)\}^2}=\frac{\sin^2\theta_1-\sin^2\theta_2}{\sin^2\theta_2-\sin^2\theta_3}=\psi(k, a^1), \tag{8}$$

[1] *Astrophysical Journal*, **35**, 333, 1912.

TABLE Ix

RELATION BETWEEN k, α^2 AND $p(k, \alpha^1)$ FOR TOTAL ECLIPSES OF DISKS COMPLETELY DARKENED TOWARD THE EDGE

α^1	k=1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2
0.00	+1.000	+1.000	+1.000	+1.000	+1.000	+1.000	+1.000	+1.000	+1.000
.02	0.796	0.800	0.804	0.808	0.812	0.816	0.820	0.823	0.827
.05	.685	.689	.694	.698	.703	.708	.712	.717	.721
.10	.543	.550	.558	.564	.569	.576	.583	.591	.599
0.15	+0.428	+0.437	+0.447	+0.455	+0.462	+0.471	+.0480	+0.492	+0.503
.20	.328	.338	.349	.359	.369	.379	.389	.402	.415
.25	.241	.251	.263	.273	.283	.295	.306	.320	.334
.30	.160	.169	.181	.192	.203	.216	.229	.244	.259
0.35	+0.081	+0.091	+0.104	+0.116	+0.128	+0.142	+0.156	+0.172	+0.187
.40	.004	.017	.032	.045	.057	.072	.085	.101	.117
.45	−0.070	−0.055	−0.039	−0.025	−0.011	.003	.018	.033	.049
.50	.143	.126	.108	.093	.078	−0.064	−0.049	−0.034	−0.018
0.55	−0.214	−0.196	−0.176	−0.161	−0.145	−0.130	−0.115	−0.100	−0.084
.60	.284	.265	.245	.229	.213	.197	.182	.167	.151
.65	.353	.334	.314	.298	.282	.265	.249	.235	.219
.70	.424	.403	.383	.367	.351	.334	.318	.304	.290
0.75	−0.495	−0.475	−0.454	−0.438	−0.423	−0.406	−0.390	−0.377	−0.365
.80	.570	.549	.528	.513	.498	.482	.467	.454	.443
.85	.650	.628	.607	.592	.578	.563	.549	.538	.528
.90	.737	.716	.697	.682	.668	.656	.644	.635	.626
0.95	−0.837	−0.818	−0.800	−0.784	−0.770	−0.761	−0.753	−0.747	−0.741
.98	.910	.889	.876	.865	.855	.850	.846	.840	.836
.99	.945	.930	.917	.907	.898	.894	.892	.888	.885
1.00	−1.000	−1.000	−1.000	−1.000	−1.000	−1.000	−1.000	−1.000	−1.000

when we follow the method of the uniform-disk theory and give three definite values to a^1 and the corresponding definite values to θ. The function $\psi(k, a^1)$, which enables us to make use of the whole light-curve in determining k, is exactly the same as before except that it depends now on the new values of the p-function. By assigning to a_2^1 and a_3^1 the permanently fixed values 0.6 and 0.9, respectively, $\psi(k, a^1)$ becomes a function of k and a_1^1 only, and by equation (8), with the aid of the p-table described above, it was readily computed for the values of k and a_1^1 entered in Table II*x*. This table corresponds to the second one of Russell's paper.

A third table, analogous to Table II*a* of the former paper, was constructed to aid in the derivation of the remaining elements of a system of completely darkened stars after k and the final light-curve have been determined in the usual way. Setting $A=\sin^2\theta_2$, $B=\sin^2\theta_2-\sin^2\theta_3$, so that, from (8),

$$\sin^2\theta_1=A+B\psi(k, a_1^1)\,, \tag{9}$$

we can derive from (7) the convenient relations

$$r_1^2 \operatorname{cosec}^2 i=\frac{B}{\phi_1^1(k)}\,, \tag{10}$$

and

$$\cot^2 i=\frac{B}{\phi_2^1(k)}-A\,, \tag{11}$$

where

$$\phi_1^1(k)=\frac{4k}{\psi(k, 0)-\psi(k, 1)} \text{ and } \phi_2^1(k)=\frac{4k}{(1-k)^2\psi(k, 0)-(1+k)^2\psi(k, 1)}\,. \tag{12}$$

These last two quantities are tabulated for convenient intervals of k.

We are now in a position to work as easily with a system of stars darkened according to our adopted law to zero at the limb as with a pair assumed to be uniformly bright, provided that the eclipse is total. The above formulae and tables complete the working theory only in those cases where there is a constant phase at minimum and the eclipsing star is the larger. During the annular phase of an eclipse of a darkened disk, the light will not be constant. As a consequence, for those systems where the secondary minimum has been observed, the relative depths of the two minima can no longer be used to determine k, as is possible on the assumption of uniform

TABLE IIx

RELATION BETWEEN k, α_1^1 AND $\psi(k, \alpha_1^1)$ FOR TOTAL ECLIPSES OF DISKS COMPLETELY DARKENED TOWARD THE EDGE

α_1^1	k=1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2
0.00	+7.865	+6.676	+5.752	+5.050	+4.476	+3.979	+3.571	+3.189	+2.853
.02	6.115	5.238	4.558	4.042	3.618	3.240	2.928	2.642	2.383
.05	5.248	4.509	3.936	3.483	3.135	2.828	2.566	2.322	2.104
.10	4.215	3.649	3.206	2.875	2.583	2.344	2.142	1.958	1.789
0.15	+3.440	+3.000	+2.651	+2.394	+2.170	+1.983	+1.820	+1.674	+1.546
.20	2.825	2.473	2.196	1.993	1.821	1.669	1.542	1.430	1.328
.25	2.314	2.034	1.810	1.649	1.513	1.395	1.298	1.210	1.130
.30	1.869	1.649	1.473	1.347	1.238	1.149	1.076	1.011	0.946
0.35	+1.475	+1.306	+1.173	+1.078	+0.994	+0.927	+0.871	+0.822	+0.772
.40	1.115	0.994	0.901	0.831	0.770	0.721	0.679	0.644	0.609
.45	0.794	0.711	0.651	0.603	0.563	0.528	0.500	0.475	0.452
.50	0.504	0.453	0.418	0.391	0.368	0.346	0.329	0.314	0.300
0.55	+0.240	+0.217	+0.201	+0.190	+0.181	+0.170	+0.163	+0.156	+0.149
.60	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
.65	−0.217	−0.199	−0.188	−0.180	−0.173	−0.165	−0.159	−0.154	−0.150
.70	0.411	0.385	0.365	0.352	0.340	0.328	0.318	0.309	0.302
0.75	−0.585	−0.556	−0.534	−0.518	−0.505	−0.490	−0.478	−0.468	−0.460
.80	0.740	0.714	0.695	0.680	0.667	0.654	0.643	0.634	0.628
.85	0.879	0.861	0.850	0.840	0.832	0.823	0.815	0.810	0.807
.90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.95	−1.097	−1.125	−1.147	−1.161	−1.173	−1.188	−1.203	−1.216	−1.227
.98	1.138	1.191	1.237	1.272	1.305	1.336	1.369	1.388	1.419
.99	1.149	1.220	1.276	1.323	1.362	1.403	1.477	1.474	1.501
1.00	−1.156	−1.257	−1.347	−1.424	−1.495	−1.559	−1.617	−1.669	−1.715

disks, and for a like reason the relation $a_0 = 1 - \lambda_1 + \frac{1-\lambda_2}{k^2}$, which for uniform brightness is of basic importance in the partial eclipse theory, fails to be valid. In addition to the failure of this previously rigorous equation on account of the dependence of the depth of the minima upon the darkening coefficient as well as upon k and the area obscured, a brief investigation has shown that the remarkable approximate relations upon which were based the empirical ω-functions, which fortunately saved the "uniform" partial eclipse problem from a long trial and error process, likewise fail to hold for darkened stars. We shall soon see, however, that the problem may be solved in another way for partial eclipses.

TABLE II*ax*

FOR COMPUTING THE INCLINATION AND THE RADIUS OF LARGER STAR

k	$\phi_1^1(k)$	$\phi_2^1(k)$
1.00	0.444	0.865
0.95	.451	.824
.90	.454	.782
.85	.454	.740
.80	.451	.698
0.75	0.444	0.656
.70	.433	.614
.65	.420	.572
.60	.403	.530
0.55	0.383	0.488
.50	.361	.445
.45	.336	.402
.40	.308	.359
0.35	0.278	0.317
.30	.246	.274
.25	.212	.230
0.20	0.176	0.186

2. *Comparison of the results obtained on the two hypotheses.*—A comparison of the two tables of the ψ-function indicates how the orbital elements will differ on the two assumptions. The solutions for an orbit will be identical up to the point where we enter the ψ-table for k. For any chosen value of the loss of light, α^1, we find that a given value of $\psi(k, \alpha^1)$ will correspond to a larger k on the assumption of darkening than for uniform brightness.

The mean "darkened" k from determinations all along the curve will be about two-tenths greater than the mean "uniform" k, depending on the magnitude of the latter. If we denote the former by k_d, and the latter by k_u, and adopt a similar notation to distinguish other quantities belonging to the two solutions, we may write the relation between them in the form

$$\psi_u(k_u, \alpha_1) = \psi_d(k_d, \alpha_1). \tag{13}$$

We may therefore regard k_u as a function of k_d and α_1 (or vice versa). Keeping k_d constant, and varying α_1, we find that the values of k_u are nearly constant except at the extreme ends of the range. A typical example is the following, in which $k_d = 0.700$:

α_1	k_u	α_1	k_u	α_1	k_u
0.00	0.670	0.40	0.506	0.95	0.538
0.10	0.572	0.50	0.495	0.98	0.544
0.20	0.546	0.70	0.480	0.99	0.514
0.30	0.528	0.80	0.473	1.00	0.450

It follows that a light-curve computed on the assumption of uniformly illuminated disks, with the mean of these values of k (which, taking account of weights, is 0.525), will agree very closely with one computed on the assumption of disks completely darkened at the edge, with $k = 0.700$. In many cases the fit can be improved by slight alterations in the other constants A and B used in computing the curves.

How good an agreement can be attained in this way can best be shown by numerical examples, of which the three that are here given are typical. The table gives the values of sin θ_1, computed from the equation $\sin^2 \theta_1 = A + B\psi\ (k, \alpha_1)$, in which B has been taken as 0.01 for the darkened stars, and A so determined that the duration of totality shall be about half its maximum value, thus presenting a typical case. The values of A, B, and k for the uniform disks have been chosen so as to get as good a representation as possible of the general course of the "darkened" curve between $\alpha_1 = 0.1$ and $\alpha_1 = 0.99$.

	Darkened	Uniform	Darkened	Uniform	Darkened	Uniform
k..........	1.000	0.850	0.700	0.525	0.400	0.200
A..........	0.01156	0.01145	0.01500	0.01500	0.01800	0.01810
B..........	0.01000	0.00980	0.01000	0.01000	0.01000	0.01020
$\sin\theta_1$ for $\alpha_1=$						
0.0	0.3003	0.2795	0.2560	0.2374	0.2317	0.2148
0.1	.2316	.2304	.2092	.2055	.1985	.1942
0.2	.1995	.2003	.1871	.1860	.1828	.1814
0.3	.1738	.1742	.1666	.1664	.1696	.1694
0.4	.1507	.1511	.1526	.1529	.1574	.1580
0.5	.1289	.1287	.1375	.1379	.1460	.1466
0.6	.1076	.1070	.1224	.1224	.1341	.1345
0.7	.0864	.0857	.1071	.1068	.1218	.1219
0.8	.0645	.0639	.0901	.0905	.1074	.1071
0.9	.0395	.0406	.0707	.0707	.0895	.0889
0.95	.0243	.0257	.0582	.0580	.0772	.0763
0.98	.0134	.0131	.0468	.0462	.0656	.0658
0.99	.0084	Max. eclipse for $\alpha_1=0.987$	.0421	.0426	.0594	.0602
1.00	.0000		.0276	.0332	.0427	.0504

Computations for other values of k give similar results, which may be summarized as follows:

Given any light-curve arising from the total eclipse of a star, completely darkened at the limb, by another (not more than three times its diameter), it is possible to find a light-curve due to the total eclipse of one uniformly bright disk by another which will agree with the former almost perfectly between obscurations of 10 per cent and 98 per cent of the whole light of the eclipsed star. The extreme phases, at the beginning and end of eclipse, will be much slower for the "darkened" curve. The light changes in the immediate vicinity of totality will be slower for the "darkened" curve unless k for this curve exceeds 0.83, when they will be more rapid.

The values of A and B for the two curves will be very nearly the same, but that of k will be much smaller for the "uniform" curve. In the extreme case of a central eclipse of equal, darkened stars (which must obviously be of very rare occurrence), the corresponding elements for uniform disks lead to an eclipse which is not quite total.

The only difference between a total and a partial eclipse is that in the latter case the constant A in equation (9) is smaller, so that $\sin^2\theta$ vanishes for some value of α_1 less than unity which we have previously called α_0. It follows at once that also in this case a "uniform" light-curve and set of elements may be substituted for the "darkened" ones, which differ from the latter only in the less gradual character of the extreme phases. The constant α_0 will be identical for the two curves; the others, A, B, and k, will be related as above.

We may proceed further and tabulate the relations between the systems of elements obtained on the two hypotheses. In doing so it is well to throw them into as general a form as possible. The previous comparison of light-curves shows that the percentage change in B, in passing from the "darkened" to the "uniform" curve, depends only on k, and that the change in A is one-half that in B. We may therefore write

$$B_u = B_d + \delta B_d\,, \qquad A_u = A_d + \tfrac{1}{2}\delta B_d\,, \tag{14}$$

where δ depends only on k. Introducing these into the equations (10) and (11), we find readily

$$\left.\begin{aligned} r_{1u}\operatorname{cosec}^2 i_u &= \alpha r_{1d}\operatorname{cosec}^2 i_d \\ r_{2u}\operatorname{cosec}^2 i_u &= \beta r_{2d}\operatorname{cosec}^2 i_d \\ \cot^2 i_u = \cot^2 i_d + \gamma B &= \cot^2 i_d + \gamma'^2 r_{1u}^2 \operatorname{cosec}^2 i_u \end{aligned}\right\} \tag{15}$$

where α, β, γ, γ' are functions of k_d and the corresponding k_u (and hence of either one alone), which may be written down at once. If i_d is to be real, $\cot^2 i_u$ must not be less than γB. Hence if we set $\gamma B = \cot^2 i_0$, i_0 is the nearest value to 90° which i_u may have, if the "uniform" curve is to be capable of representation by a "darkened" curve with the assigned value of k_d. We may then put the last of equations (15) in the form

$$\left.\begin{aligned} \cot^2 i_u &= \cot^2 i_d + \cot^2 i_0 \\ \cot i_0 &= \gamma' r_{1u}\operatorname{cosec} i_u \end{aligned}\right\} \tag{16}$$

The quantities α, β, γ', δ are tabulated below. With their aid it is possible (within certain limits) to pass at once, by means of equations (15) and (16), from any given set of elements derived on one

hypothesis to one which, on the other hypothesis, will give an almost identical light-curve (with the small differences already specified).

The depth of the secondary minimum, computed from these two sets of elements, will be different. If L_1 is the light of the larger star (which at principal minimum eclipses the other), and a'' denotes the fraction of this which is obscured at secondary minimum, the loss of light at this time is $a''L_1$. If one star is completely projected on the other's disk, we have for uniform disks $a''_u = k^2_u$. For darkened disks a'' depends on the apparent position of the smaller disk in front of the larger, being greatest when it is central—then $a''_d = 1-(1-k^2_d)^{\frac{3}{2}}$—and least when it is internally tangent (when computation by quadratures is necessary). These values are given in the table under the headings a''_d and a'''_d.

TABLE C

FOR THE TRANSFORMATION OF "UNIFORM" INTO "DARKENED" ELEMENTS, OR VICE VERSA

k_d	1.000	0.900	0.800	0.700	0.600	0.500	0.400
k_u	0.850	0.735	0.630	0.525	0.420	0.310	0.200
a	1.019	1.027	1.040	1.065	1.107	1.168	1.283
β	0.865	0.840	0.821	0.798	0.775	0.724	0.642
γ'	0.180	0.254	0.314	0.366	0.431	0.528	0.650
δ	−0.020	−0.010	0.000	0.000	0.000	+0.010	+0.020
a''_u	0.722	0.541	0.397	0.276	0.176	0.096	0.040
a''_d	1.000	0.917	0.784	0.636	0.488	0.350	0.229
a'''_d	1.000	0.904	0.750	0.581	0.427	0.289	0.175

Various interesting conclusions can be drawn from this table. From the values of a, β, γ', it appears that the "uniform" solution always gives the large star larger, the small star smaller, and the inclination farther from 90° than the "darkened" solution; and hence if darkening really exists, the solution made without regarding it will lead to a density too low for the large star, too high a density for the smaller one, and too great a ratio of the surface brightness of the two (in the case here discussed where the faint star is the larger). The most important effect is on the computed density of the smaller star, which on the average will be about twice too great.

The numerical data of a paper which will appear soon[1]

[1] In the *Astrophysical Journal* for November, 1912.

show that the difference in form between the light-curves at principal minimum derived on the two hypotheses is almost at the limit of detection, even by the most numerous and accurate observations so far published, and is certainly too small to justify the hope of distinguishing between different degrees of darkening toward the limb.

The only chance of this seems to be by means of the secondary minimum. It appears from Table C that, in the case of complete darkening at the limb, the depth of secondary minimum, corresponding to a given form of the light-curve of primary minimum, will exceed the corresponding depth in the case of uniform disks by about three-tenths of the light of the fainter star. Even with a range of $2^{mg}.5$ at principal minimum, the secondary should be $0^{mg}.03$ deeper on the first hypothesis than on the second. With a range at principal minimum of $1^{mg}.0$ the difference would be $0^{mg}.13$. The secondary minimum lasts as long as the primary, and, if equally well observed, its depth should be determinable within less than $0^{mg}.01$—provided that the non-eclipse part of the curve has also been so well observed as to determine and eliminate the ellipticity and radiation effects. It should therefore be possible in this way to determine conclusively, not only whether certain stars are darkened toward the limb, but the extent of the darkening. The best stars for the purpose are those with a total eclipse of relatively small depth at primary minimum. Unfortunately, no sufficient data for the whole light-curve of such a star are at present available.

Unless the observations upon which a star's light-curve is based are of unusual number or accuracy, the transition from the elements computed on the assumption of uniform disks to those which assume complete darkening at the limb may be made with the aid of Table C. When the light-curve is very accurately determined, a direct solution by means of Table IIx is preferable. This may give results differing slightly from those of Table C, as the detailed solution will give a light-curve which fits the observed curve most closely where the course of the latter is determined with the greatest weight, while the comparisons upon which Table C is based aim at obtaining a good general fit all over the curve.

One exceptional case needs further discussion. If the value of

TABLE IV

VALUES OF $\Phi(k, \alpha_1)$ FOR UNIFORMLY ILLUMINATED DISKS

For Computing the Values of k and the Light-Curve in the Case When the Eclipse Is Central. $\sin^2 \theta = \Phi(k, \alpha_1) \sin^2 \theta_6 = A\Phi(k, \alpha_1)$

α	k=1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2
0.00	9.887	7.345	5.683	4.500	3.608	2.920	2.368	1.925	1.556
.10	6.407	4.922	3.940	3.233	2.695	2.266	1.919	1.627	1.391
.20	4.668	3.700	3.047	2.572	2.209	1.914	1.673	1.472	1.294
.30	3.368	2.761	2.363	2.062	1.826	1.635	1.470	1.332	1.210
.40	2.399	2.056	1.822	1.648	1.522	1.398	1.300	1.215	1.136
.50	1.609	1.471	1.371	1.299	1.239	1.189	1.144	1.104	1.067
.60	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.70	0.549	0.631	0.692	0.744	0.789	0.829	0.870	0.901	0.935
.80	0.244	0.351	0.443	0.525	0.599	0.670	0.738	0.804	0.869
.90	0.061	0.152	0.245	0.337	0.428	0.518	0.610	0.702	0.798
.95	0.019	0.080	0.151	0.248	0.341	0.438	0.540	0.645	0.751
.98	0.002	0.045	0.112	0.191	0.281	0.381	0.490	0.604	0.727
1.00	0.000	0.020	0.070	0.140	0.225	0.324	0.436	0.558	0.692

TABLE IVx

VALUES OF $\Phi(k, \alpha_1)$ FOR DISKS COMPLETELY DARKENED TOWARD THE EDGE

For Computing the Values of k and the Light-Curve in the Case of Central Eclipse

α	k=1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2
0.00	7.805	6.220	5.015	4.100	3.373	2.770	2.281	1.874	1.530
.10	4.648	3.856	3.238	2.763	2.369	2.044	1.769	1.536	1.333
.20	3.446	2.917	2.532	2.223	1.966	1.742	1.554	1.392	1.247
.30	2.617	2.290	2.028	1.826	1.656	1.512	1.386	1.277	1.176
.40	1.965	1.778	1.629	1.510	1.408	1.321	1.244	1.177	1.113
.50	1.435	1.354	1.292	1.240	1.195	1.154	1.118	1.086	1.056
.60	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.70	0.644	0.698	0.746	0.784	0.820	0.854	0.886	0.915	0.944
.80	0.360	0.442	0.515	0.583	0.646	0.709	0.769	0.826	0.883
.90	0.135	0.218	0.302	0.386	0.470	0.555	0.641	0.726	0.814
.95	0.051	0.120	0.200	0.287	0.378	0.471	0.568	0.667	0.772
.98	0.016	0.069	0.137	0.219	0.309	0.406	0.509	0.620	0.736
1.00	0.000	0.017	0.061	0.127	0.209	0.307	0.419	0.544	0.681

cot i determined on the assumption of uniform disks is less than cot i_0 (as defined by equation (16) and Table C), there will be no real "darkened" solution with the value of k_b resulting from this table, and it will be necessary to represent the observations, as well as may be done, by a set of elements for which cot $i=0$, i.e., on the hypothesis of central transit. In this case, by (9) and (11), $B=A\phi_2(k)$, $\sin^2\theta=A\{1+\phi_2(k)\psi(k, a_1)\}$. To simplify the process of getting the best fit for the observed curve under these circumstances, we may set $1+\phi_2(k)\psi(k, a_1)=\Phi(k, a_1)$ and tabulate this function. Then $\sin^2\theta=A\Phi(k, a_1)$, and the solution is made with the aid of the new table, just as in the general case with the ψ-table. The computation of the elements is immediate, for $i=90°$, and, if θ' and θ'' correspond to the beginning of eclipse and of totality,

$$r_1=\tfrac{1}{2}(\sin\theta'+\sin\theta'')\ ,$$
$$r_2=\tfrac{1}{2}(\sin\theta'-\sin\theta'')\ .$$

As no similar table has so far been published, the Φ-function is tabulated here both for uniform disks and for those completely darkened at the limb.

The full discussion of total eclipses of darkened stars is now completed. When elements have been derived for a partial eclipse on the hypothesis of uniform disks, and the larger star is found to be in front at principal minimum, Table C makes it possible to find at once (for values of k within its range) a set of "darkened" elements which will give sensibly the same light-curve for this minimum, except for the greater extent of the extreme phases at beginning and end. These elements will, however, give a decidedly deeper secondary minimum than the corresponding "uniform" elements.

It is obvious that by computing "uniform" elements for the same principal minimum, but a shallower secondary, a set of "darkened" elements can be found which will represent both observed minima; but the detailed discussion of this, and of annular eclipses, must be postponed to another communication to avoid undue delay of the present publication.

PRINCETON UNIVERSITY OBSERVATORY
August 6, 1912

ON CONVERGENCE FREQUENCY IN SPECTRAL SERIES

By FERNANDO SANFORD

It is the purpose of this paper to show some relations between the convergence frequencies (A, in Kayser and Runge's formula) in the spectral series of certain atoms and some of their other physical properties.

Professor Kayser says in his *Handbuch der Spectroscopie*, Band II, S. 592:

> Man sollte erwarten, da die Lage des Spectrums sich vom Atomgewicht abhängig erweist, dass z. B. auch die erste Constante in einer mathematisch ausdrückbaren Beziehung zum Atomgewicht stehen werde. Aber es ist bisher nicht gelungen, eine solche Beziehung zu finden; man kommt hier nicht weiter, als bis zum Ausspruch, dass die Lage des Serienspectrums eine periodische Function der Atomgewichte sei, ebenso, wie man für die übrigen Eigenschaften der Elemente nicht weiter gelangt ist.

I am not aware that such a relation has been observed since the above was written, but I am not familiar enough with the literature of spectroscopy to know whether it has or not. However, I am sure that the particular relation to which I wish first to call attention has not been observed.

Since the quantities to which I refer, and which I have called the characteristic charges of the atoms, are not generally recognized, it will be necessary for me to explain briefly what I mean by the term, and how these quantities are determined.

In a paper entitled "On Positive Atomic Charges," published in *Physical Review*, **32**, 512, May 1911, and in a paper entitled "A Physical Theory of Electrification," *Leland Stanford Junior University Publications, University Series*, No. 6, I gave reasons for believing that the positive sub-atoms of the elements have characteristic charges which are not simple multiples of the charge on a positive hydrogen atom. An attempt was made to calculate the relative magnitude of these charges on the ions in electrolysis on the assumption that these ions are dissociated for only very brief

periods and move but short distances in the solution before recombining with another ion or with a water molecule. Under these conditions their motion through the solution will be accelerated according to the equation $F=ma$, where F is the electromotive force acting upon the ion, and m is its ionic mass. Since the force acting upon an ion in a given electric field is proportional to its charge, we may substitute e, the ionic charge, for F in the above equation. The charges of different ions in a given electric field (assuming them to be free for the same periods of time) would be proportional to the products of their ionic masses into their ionic speeds.

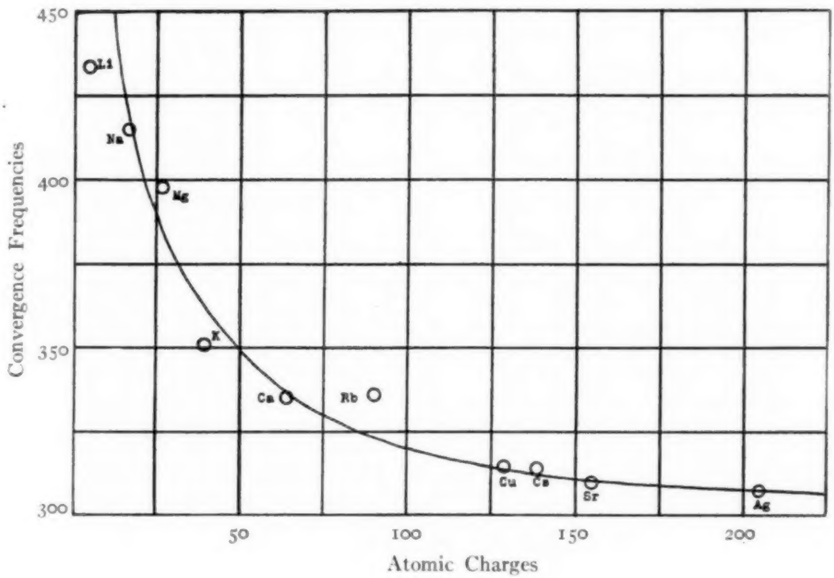
I have calculated in this way the *relative* charges of those atomic ions whose velocities in an electrolytic solution have been determined. Assuming the absolute charge of the hydrogen ion to be 4.84×10^{-10}, I have then calculated the corresponding characteristic charges of the other ions. In the papers referred to above I have shown that the numbers obtained in this way are related to all the properties of the ions which determine cohesion or affinity, and that they are likewise arranged in the order of their atoms in the voltaic series.

In the table below are given the values of these quantities for all the atoms for which I have been able to calculate them by the above method and for which the convergence frequencies are given in the table on p. 593, Vol. II, of Kayser's *Handbuch*. The convergence frequencies are in all cases the highest given in the table; that is, they are taken from the principal series in the case of the alkali metals and from the first subordinate series in the case of the other metals used.

Element	Characteristic Charge	Convergence Frequency
Li	3.96	43584
Na	15.8	41543
K	38.8	35091
Rb	89	33762
Cs	138	31509
Mg	26.4	39796
Ca	69.6	33919
Sr	155	31030
Cu	128.4	31592
Ag	205	30712

In the case of the bivalent elements the charges calculated for a single valence (for which the ionic velocities are given in the tables) are multiplied by two. The charges are all multiplied by 10^{10}.

To show the relations existing between the numbers in the two columns, they are plotted against each other in Curve I, the atomic charges being used as abscissae and the convergence frequencies as ordinates. Only three decimal places of the convergence frequencies are used. The hyperbola drawn in the figure is calculated from



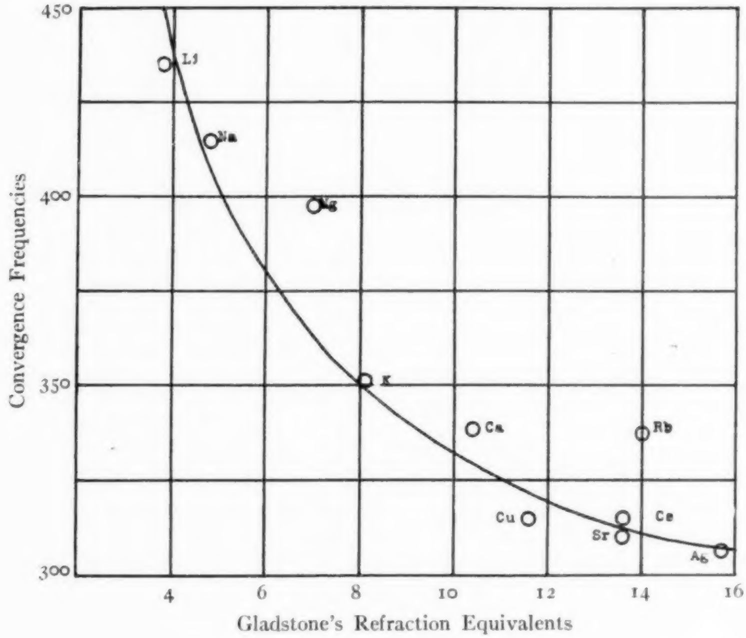
CURVE I

the equation $(e+5)\ (f-295)=2840$. It is not claimed that this is the correct equation for the relation between the two sets of quantities, or even that it is the nearest possible approximation to it, but only that it does approximate to the relation under consideration.

If a similar curve be plotted using the first subordinate series of the alkali metals, it will be found that the convergence number for hydrogen falls upon this curve.

That Curve I does show a physical relation between the convergence numbers and the atomic charges calculated in this way

seems beyond a doubt. The significance of this relation seems to me to lie in the fact that when the above elements are arranged in the order of magnitude of their atomic charges they are also arranged in their order in the voltaic series. Coehn's law indicates that the voltaic series is the same as the dielectric series, and that the metals when arranged in the order of the voltaic series are also arranged in the order of their specific inductive capacities. I have



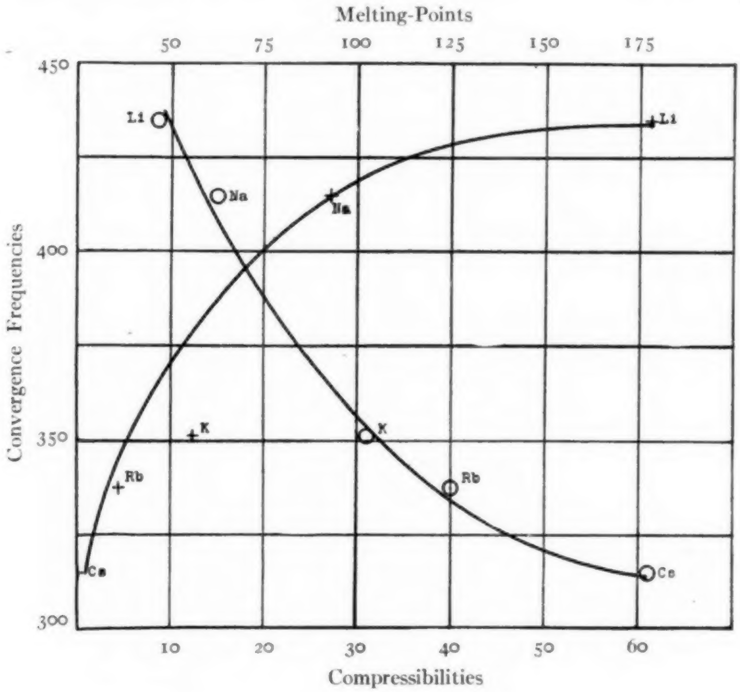
CURVE II

elsewhere[1] given what seem to me to be valid reasons for expecting this relation between the voltaic series and the dielectric series. If this relation holds in metals, as it does in non-metals, the above table shows that the convergence frequency is greater in those elements having the lower specific inductive capacity. This seems reasonable, since the convergence frequency apparently represents the highest attainable frequency of oscillation of any electron in a given system. If these electrons vibrate under the influence of a

[1] "Physical Theory of Electrification," pp. 41–43.

central electric force, there is no question that, other things being equal, the higher the specific inductive capacity of the system the weaker the central force and the slower the oscillation.

In an article on "Pressure-Shift of Spectral Lines" in this *Journal,* January 1912, I have already called attention to the relation between the refraction constants of some of the elements and their atomic charges. A similar relation between the refraction con-



CURVE III

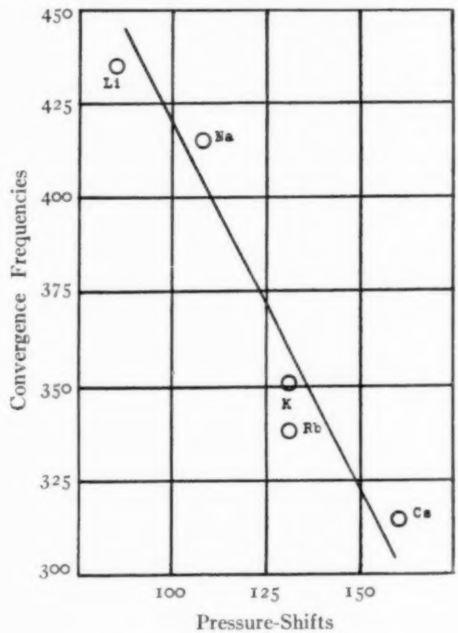
stants and convergence frequencies of the elements in the above table is shown in Curve II, where Gladstone's refraction constants[1] are taken as abscissae and the convergence frequencies as ordinates. The abscissae are so chosen that the two curves shall be on nearly the same scale.

The curve shows that the refraction constant varies in the inverse direction as the convergence frequencies. It seems to be

[1] *Phil. Trans.*, 1870, pp. 23–26.

pretty definitely established that the inductive capacity of a medium is a principal determining factor of the light-velocity in that medium, and hence of its refraction constant.

In the paper on pressure-shift mentioned above, I also called attention to the relation of this property to the compressibility and the melting-points of the alkali metals. The relation between these properties and the convergence frequencies is shown in Curve III. · In Curve III the points represented by crosses indicate the



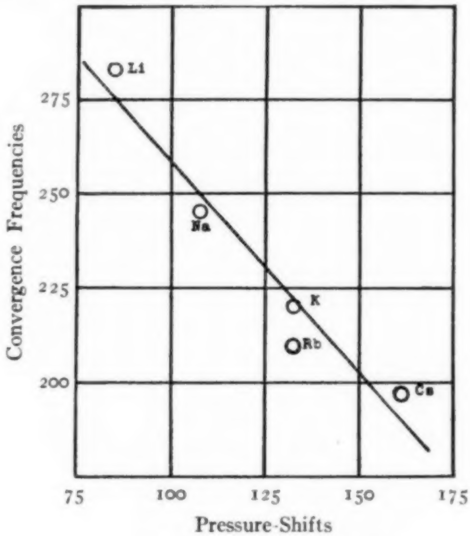
CURVE IV.—Principal series

melting-points, the values of which are given at the top of the curve, while the circles represent the compressibilities as determined by Richards.[1]

If the internal forces of the atoms are electrical forces, and if the forces which hold together the atoms in a metal are of the same nature, then both the compressibilities and the melting-points must depend upon the inductive capacity of the atoms.

[1] *Zeit. Phys. Chem.*, **61**, 192, 1910.

It follows from the relations which have been shown that the pressure-shift and the convergence frequencies of the elements of this group must bear an inverse relation to each other. In Curves IV and V these relations are shown for the convergence frequencies of the principal series and the subordinate series of the alkali metals. Whether, if the lines of these different series were considered separately, their pressure-shifts would be found to differ, I do not know.



Curve V.—Subordinate series

In his reply to my article on pressure-shift, Humphreys dismisses my arguments with the statement that pressure-shift is equally well related to all the other properties of the atom which are periodic functions of the atomic weight. This is no doubt true in a mathematical sense. It would be equally true if the pressure-shift varied in the inverse, instead of the direct, order of the specific inductive capacity. It is, however, a matter of importance to one who attempts to *explain* phenomena by mechanical laws, rather than to find rules for computing their magnitude, to know in what direction with reference to each other they vary. It seems to be plainly shown that in one group, at least, the pressure-shift varies

in the inverse direction to the forces which hold together the electrical parts of the atom and the atoms of the solid. If this shift is proportional to the magnetic fields of the atoms when luminous, it follows that those atoms which have the strongest magnetic fields when luminous are those which attract each other with the weakest forces when not luminous. I see no physical reason why this should be the case. Neither do I see why those atoms whose electrons are capable of rotating at the highest speeds when luminous (if we assume the Saturnian atom) should have the weakest magnetic fields, also when luminous.

STANFORD UNIVERSITY
July 1912

Reviews

Die optischen Instrumente: Aus Natur und Geisteswelt, Sammlung wissenschaftlichgemeinverständlicher Darstellungen 88stes Bändchen. Von Dr. Moritz von Rohr. Leipzig: B. G. Teubner, zweite, vermehrte und verbesserte Auflage, 1911. Pp. vi+140. Bound, M. 1.25.

Dr. von Rohr's book, which appears now in its second edition, aims to state clearly and accurately the principles underlying the construction and operation of the better known forms of optical instruments. In the second edition new material has been introduced at several points and the whole book seems to be printed in a new style of type, much easier on the eyes than was that of the first edition. The arrangement and subject-matter are largely the same in both editions—lenses, the human eye, binocular vision, the photographic camera, the stereopticon, the cinematograph, eyeglasses, reading-glasses, the microscope, and the telescope being taken up in order. The discussion of the microscope is especially interesting. The use of prisms for the purpose of total reflection is shown in the case of several styles of binoculars. The prism as a dispersing medium and its use in simple refraction, as well as its use in the treatment of strabismus, might have been mentioned.

Some care has been taken in the discussion of the different kinds of aberration in lenses, and a short account of the different types of lenses and their historical development round out the little book, of which the small size has made it necessary for the author to condense much that might have been written more at length.

Dr. von Rohr is well known through his book on the *Theory and History of the Photographic Objective* and through his frequent contributions to the *Zeitschrift für Instrumentenkunde.* A short time ago Czapski secured his help on the second edition of *Die Theorie der optischen Instrumente;* and Dr. von Rohr is publishing, in collaboration with the other scientific assistants of the great Zeiss firm of Jena, a series of volumes on the theory of optical instruments. This series, when complete, will probably surpass all other works on the subject, and will be a

stimulus, as its first volume has already been, to the production of treatises and books on optics, more nearly complete and more valuable than have previously appeared in English.

The little book of Dr. von Rohr's is, on the other hand, popular in style, not theoretical but descriptive. It can be depended on as far as it goes.

STANLEY C. REESE

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